Order and chaos in spiral galaxies
(Heinrich Eichhorn Lecture)

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Abstract Spiral galaxies contain both ordered and chaotic orbits. In normal spirals the perturbations are weak (of order 2–10%) and most orbits are ordered. The density wave theory refers mainly to linear perturbations. Nonlinear effects appear in the outer parts of the open spirals (S_b, S_c) and produce the termination of these spirals near the 4/1 resonance. On the other hand in barred spirals the perturbations are relatively strong (of order 100%). Then the outer spirals and the envelope of the bar are composed mainly of chaotic orbits, while the main body of the bar is composed of ordered orbits. The chaotic orbits of the spiral arms of strong barred galaxies are sticky, i.e. they do not escape from the galaxy for at least a Hubble time. The forms of these spirals are delineated by the unstable manifolds of the unstable periodic orbits L_1, L_2 near the ends of the bar and of other unstable periodic orbits inside and outside corotation.

Keywords Density wave theory · Spiral galaxies · Non-linear effects · Barred galaxies · Velocities · Long-time evolution · Spiral density waves · Sticky chaotic orbits

1 Introduction

I am honored and happy to give this lecture in memory of Heinrich Eichhorn. Heinrich Eichhorn was a pioneer in Dynamical Astronomy. His work covered many fields, mainly in kinematics and Celestial Mechanics, but also in general dynamical problems and on philosophical and epistemological questions. He played an important role in developing the Astronomy Department of the University of Florida. We organized a series of Symposia in Dynamical Astronomy, that brought together the best people in this field. The proceedings of most of these Symposia were published by the New York Academy of Sciences and they are used extensively up to now by the specialists. One of his main interests was order and chaos in Astronomy. Thus it is appropriate that I speak today on the most recent developments in the
area of order and chaos in spiral galaxies, emphasizing my own work on this subject. An earlier review on these problems can be found in my book on Order and Chaos in Astronomy (Contopoulos 2002).

The orbits in spiral galaxies are either ordered or chaotic. The general form of ordered orbits is shown in Fig. 1a. Such orbits are quasi-periodic. They consist of epicyclic oscillations superposed on periodic orbits around the center, which are like ellipses. On the other hand chaotic orbits fill all the space inside a curve of zero velocity (Fig. 1b).

However there are also ordered orbits around higher order periodic orbits closing after more than one revolution around the center (Fig. 2a) or around periodic orbits surrounding the stable Lagrangian points L4 and L5 (ring orbits and banana orbits Fig. 2b). On the other hand there are chaotic orbits that form rings around the center (Fig. 3a) or double rings, surrounding the center and the Lagrangian points L4 and L5 (Fig. 3b). Finally there are also orbits that escape to infinity (Fig. 4).

In a rotating frame of reference with angular velocity \( \Omega \) the Jacobi constant

\[
H = E - \Omega J
\]  

(1)

(where \( E \) is the energy and \( J \) the angular momentum) is conserved. But in chaotic orbits neither \( E \) nor \( J \) are conserved. The orbits may go to large distances (Fig. 4a) following elliptic-like trajectories, with almost constant \( E \) and \( J \), but whenever they come close to the main body of the galaxy, their energy and angular momentum change in an irregular way. In Fig. 4b we mark the successive changes of the energy of an orbit that goes to different distances from the center but returns close to it until its energy becomes positive (\( E > 0 \)) and then the orbit escapes to infinity along a hyperbola-like orbit (Fig. 4a).

The above classification applies to orbits in a fixed potential, either in the inertial frame, or in a rotating frame. If the potential changes in time the orbits also change and an ordered orbit may become chaotic, or vice-versa. Such phenomena appear in N-body simulations of galaxies where we have in general an evolving potential. In such cases the best method to follow the evolution is by freezing the potential at particular times and then calculate orbits in the frozen potential. If this is done we may see the changes of the topologies of the orbits from time to time.

The study of orbits is essential in understanding the basic characteristics of the spiral galaxies.