Averaged rotational dynamics of an asteroid in tumbling rotation under the YORP torque

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Abstract The secular effect of YORP torque on the rotational dynamics of an asteroid in non-principal axis rotation is studied. The general rotational equations of motion are derived and approximated with an illumination function expanded up to second order. The resulting equations of motion can be averaged over the fast rotation angles to yield secular equations for the angular momentum, dynamic inertia and obliquity. We study the properties of these secular equations and compare results to previous research. Finally, an application to several real asteroid shapes is made, in particular we study the predicted rotational dynamics of the asteroid Toutatis, which is known to be in a non-principal axis state.

Keywords Asteroid · Rotational dynamics · YORP effect · Tumbling rotation · Asteroid: Toutatis

1 Introduction

The aim of this work is to study the rotational dynamics of a celestial body under the YORP torque. The YORP (Yarkovsky–O’Keefe–Radzievskii–Paddack) torque is the consequent effect to infrared thermal emission from the surface of a body irradiated by the Sun, and was first introduced in Rubincam (2000). This effect is mainly important for the rotational dynamics of small solar system bodies, particularly asteroids of size <10 km. Specifically, it can have a strong influence on the evolution of the obliquity and of the rotation rate of these bodies, and it may have also a role in the creation of binary asteroids.

The dynamics of the YORP effect has been previously studied by many authors and in different ways. With numerical models by Capek and Vokrouhlicky (2004) and Vokrouhlicky and Capek (2002). An analytical description of the effect is given, for example, in Nesvorny...
and Vokrouhlicky (2007), in Scheeres and Mirrahimi (2008) and more recently in Mysen (2008). As the YORP torque is a small torque, it can be treated as a perturbation to the unperturbed rotational motion, and the important effects are then secular ones over long time spans. Thus the analytical solutions given in these papers are basically first order, in the small quantity YORP torque over angular momentum, averaged solutions over the rotational and the orbital motion of the asteroid, and they are based on the hypothesis of principal axis rotation around the maximum moment of inertia.

However, there are examples of asteroids which are not in principal axis rotation, said to be in tumbling rotation, and in such cases the previous analytical models cannot be applied (see Hudson and Ostro 1995). The first work dedicated to the study of these rotators was (Vokrouhlicky et al. 2007), in which a full numerical integration of the equations of motion is performed. In this work we derive a first analytical theory that can explain the secular evolution behaviour of a non-principal axis rotator.

In Neishtadt et al. (2003) and Sidorenko et al. (2008) the generic evolutionary equations for an outgassing comet nucleus rotation were developed. Here we will follow a similar approach, looking for the averaged first order equations of a generic rigid body under the thermal YORP torques, without the hypothesis of principal axis rotation.

In addition to the so called “Rubincam’s approximation”, which consists of assuming zero thermal conductivity for the surface of the body, there are some others limitations in our analytical approach, in particular in modelling the illuminated-shaded portion of the body’s surface hit by the Sun. We should continue to use general, non-convex bodies in our studies but note that we make the strong approximation that there is no self-shadowing—i.e., model them as locally convex. Thus this work does not pretend to be a complete analytical theory of tumbling rotators under YORP, but is a first step in this direction.

The solution of the evolutionary equations that we find turns out to be easily describable in terms of few suitable quantities. The results and the conclusions that we find here include and are in agreement with the previous analytical results about principal axis rotators, but they are only partially in agreement with the numerical results in Vokrouhlicky et al. (2007), although they have in common all of the main features seen in that work. Better approximations are needed and we briefly discuss some possible generalizations of the theory in a qualitative way.

In Sect. 2 we describe the full differential equations of the rotational dynamics of a rigid body under a torque, and we define the fundamental reference systems. We describe the equations for the YORP torque and for the energy dissipation dynamics. Under the assumption of no resonance between the rotation of the body and its orbital motion, in Sect. 3 we obtain the first order (in the small parameter YORP torque over angular momentum) averaged equations for the angular momentum vector $\mathbf{L}$ of the asteroid and for the so called dynamic inertia $I_D$, which is a parameter related to the non-principal rotation mode of the body. In Sect. 4 we discuss the dynamics without energy dissipation for the angular momentum vector and the dynamic inertia along with some important generalizations. In Sect. 5 we describe the chaotic behaviour of the rotational dynamics when the body is close to the critical solution $I_D = B$, where $B$ is the intermediate moment of inertia. Finally in Sect. 6 we apply our analytical results to some real asteroid shape models.

## 2 Rotational dynamics

Our dynamical problem is the rotation of a rigid body under a torque. In order to write the differential equations that describe this dynamics, we have to define some suitable reference systems.