The low-energy physics of strong interactions suffers from the lack of a perturbation expansion of QCD in this regime. The use of phenomenological models is therefore unavoidable. The models should be based on general symmetry properties of QCD. A $SU(3)$ chiral model for the description of hadronic properties as well as of nuclei and hypernuclei is presented.

1 Introduction

Strong interaction physics enters in a number of subbranches of nuclear and high-energy physics. The strong interactions generate most of the observed elementary particles, namely the hadronic states as bound states of quarks, and most likely also glueball states (although glueballs have not yet clearly been experimentally identified). But strong interactions are also needed in standard nuclear physics to describe the properties of ordinary nuclei and, including hyperonic degrees of freedom, of hypernuclei. In addition, especially in the field of relativistic heavy-ion physics, strong interactions are tested in the extreme regimes of high densities and temperatures.

One major problem of the theoretical treatment of strong interactions is that generally all the before mentioned fields use a different model approach to describe the respective phenomena, although the accepted theory of strong interactions, namely quantum chromodynamics (QCD) governs the dynamics of the systems in all these cases.

The reason for this theoretical deficiency is based on the properties of QCD itself. Looking at the coupling strength for one-gluon exchange between two quarks we get the following result for the running coupling $\alpha_{\text{QCD}}(q^2)$:

$$\alpha_{\text{QCD}}(q^2) \sim \frac{12\pi}{(33 - 2N_f) \log(|q^2|/\Lambda^2)}$$  

(1)

$N_f$ is the number of quark flavors, $N_f = 2$ for the light quarks (up, down) and $N_f = 3$ if one includes the strange quark. The scale parameter $\Lambda$ is of the order of 200 MeV. As is evident from formula (1), which includes lowest-order QCD quantum corrections, the coupling strength becomes small at very high energies. This is the so-called asymptotic freedom of QCD which implies that for high-energy processes QCD is a rather weakly interacting theory and one can use standard techniques based on perturbation theory, very much like it is the case for quantum electrodynamics. This feature also explains the great phenomenological success of QCD

in describing high-energy strong-interaction experiments. In the realm of low-lying hadronic states, or even nuclear properties, however, the energies under consideration are too low for these methods to be applicable. One can see from Eq. (1) that the simple formula diverges for $|q| = \Lambda$ leading to large coupling strengths in the region of low momenta and energies of, let’s say $E < 1$ GeV. When the coupling strength is of the order of 1, perturbation theory is not valid anymore, as many-gluon exchange processes are not suppressed compared to one-gluon exchange. Thus, one would need to calculate an infinite number of Feynman diagrams. This central problem of low-energy QCD forces the theorist to either introduce severe model assumptions or consider direct numerical approaches. The latter one, the so-called lattice-gauge theory calculations, have become quite popular in recent years as faster computers allow for more and more extensive numerical calculations. In this approach the basic equations of QCD are discretized on a finite space-time lattice and solved directly on the computer within a path-integral description. Whereas this calculational scheme is quite successful in describing basic properties of the low-lying hadronic states like their mass spectrum, one is restricted to few-quark systems. Systems like finite nuclei or nuclear matter with finite chemical potential are far beyond reach of a numerical calculation, at least for the moment.

2 Chiral symmetry

Following the previous arguments it is clear that one is forced to rely on some model description of QCD and strong interaction physics in general. Therefore the problem arises, mentioned in the beginning, that one has a set of different models for the various low-energy systems. One more general viewpoint of approaching the modeling of strong interactions is to study its basic symmetries and to try to preserve them in a model approach. Let us focus on one important symmetry, the so-called chiral symmetry. If we consider a fermion field (let us say a quark or electron field), we can write an operator $P(n)$, projecting the fermion spinor on the spin direction parallel to a space-like vector $n^\mu$, $n^2 = -1$:

$$P(n) = \frac{1}{2}(\mathbb{1} + \gamma_5 n^\mu) .$$

(2)

Picking out the specific spin direction parallel to the momentum $\vec{k}$ of the particle we can define the corresponding vector $n_\mu$ with

$$n_\mu = \left( \frac{|\vec{k}|}{m}, \frac{k_0}{m}, \frac{\vec{k}}{m|\vec{k}|} \right)$$

(3)

with the mass of the particle $m$, where obviously $\vec{n} \parallel \vec{k}$ and the coefficients are chosen in a way to preserve the normalization. Taking the limit of massless particles, with some care, we get $m \to 0, n_\mu \to k_\mu/m$ and the projection operator reduces to

$$P_\pm = \frac{1}{2}(\mathbb{1} \pm \gamma_5)$$

(4)