FUNCTION SPACES ON $\tau$-CORSON COMPACTA AND TIGHTNESS OF POLYADIC SPACES

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Abstract. We apply the general theory of $\tau$-Corson Compact spaces to remove an unnecessary hypothesis of zero-dimensionality from a theorem on polyadic spaces of tightness $\tau$. In particular, we prove that polyadic spaces of countable tightness are Uniform Eberlein compact spaces.

Keywords: boolean, polyadic, function space, Corson, compact, $C_p(X)$, Eberlein, tightness

MSC 2000: 54D30, 54C35

0. Introduction

All of our spaces are assumed to be completely regular. For an infinite cardinal $\kappa$, let $A_\kappa = \kappa \cup \{\infty\}$ be the one point compactification of the discrete space $\kappa$. For a cardinal $\lambda$, let $A_\kappa^\lambda$ be the product of $\lambda$ copies of $A_\kappa$ endowed with the product topology. Polyadic spaces, introduced by Mrowka [15], are the continuous images of the spaces $A_\kappa^\lambda$.

The main goal of this paper is to remove the assumption of zero-dimensionality from the hypothesis of the following result by Bell

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Result 0.1 [4]. If $X$ is a zero-dimensional polyadic space of tightness $\tau$ and cellularity $\mu$, then there exists a closed $F \subset A_\mu^\tau$ such that $F$ continuously maps onto $X$.

and so get a useful structure theorem for arbitrary polyadic spaces. Motivation for Result 0.1 came from the problem in Gerlits [8] of whether every polyadic space of tightness $\tau$ and cellularity $\mu$ is a continuous image of $A_\mu^\tau$; we now know (Bell [5]) that this problem has a negative answer—there is a zero-dimensional polyadic space of countable tightness which is not an image of $A_\mu^\tau$, for any $\mu$.

Our main problem (of removing zero-dimensionality) belongs to a general class of problems that can be described as follows: For which classes $\mathcal{C}$ of compact spaces is it true that for every $X \in \mathcal{C}$, there exists a zero-dimensional $K \in \mathcal{C}$ such that $X$ is an image of $K$? That is, when is the family of zero-dimensional members of $\mathcal{C}$ mapping-universal for $\mathcal{C}$? In our case, to solve our problem, we need to develop the general theory of $\tau$-Corson Compact spaces. Since the core of our polyadic result is valid for a larger class of spaces—the continuous images of $\tau$-Valdivia compact spaces (see Section 2 for a definition), we present our main result as a corollary to a Valdivia result.

When the tightness is countable, there is the following important result of Benyamini, Rudin and Wage which we shall relate to.

Result 0.2 [6]. $X$ is a Uniform Eberlein compact space $\iff$ there exists a cardinal $\kappa$ and a closed $F \subset A_\kappa^\tau$ such that $F$ continuously maps onto $X$.

Let us recall that a space $X$ is a Uniform Eberlein compact space if $X$ is homeomorphic to a weakly compact subset of a Hilbert space.

1. Boolean preliminaries

We denote the algebra of all clopen subsets of a space $X$ by $\text{CO}(X)$. For a collection $\mathcal{C} \subset \text{CO}(X)$, put $\langle \mathcal{C} \rangle$ equal to the subalgebra of $\text{CO}(X)$ generated by $\mathcal{C}$. A generating family for $\text{CO}(X)$ is a $\mathcal{C} \subset \text{CO}(X)$ such that $\langle \mathcal{C} \rangle = \text{CO}(X)$.

A Boolean space is a compact space $X$ which has $\text{CO}(X)$ as a basis. If $\mathcal{B}$ is a boolean algebra, then $\text{st}(\mathcal{B})$ is the Stone space of all ultrafilters of $\mathcal{B}$ which uses $\{B^+ : B \in \mathcal{B}\}$ as a basis where for $B \in \mathcal{B}$, $B^+ = \{p \in \text{st}(\mathcal{B}) : B \in p\}$. If $\mathcal{C}$ is a subalgebra of $\mathcal{B}$ then the Stone map $\alpha: \text{st}(\mathcal{B}) \to \text{st}(\mathcal{C})$ is defined by $\alpha(p) = p \cap \mathcal{C}$ and is the canonical continuous surjection.

Given a set $A$ and a cardinal $\tau$, we denote by $\Sigma_\tau(R^A)$ ($\Sigma_\tau(2^A)$) the subspace of the product $R^A$ ($2^A$) consisting of all points $x \in R^A$ ($x \in 2^A$) such that $|\{a \in A: x(a) \neq 0\}| \leq \tau$. 900