ON REPRESENTATIONS OF RESTRICTED LIE SUPERALGEBRAS

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Abstract. Simple modules for restricted Lie superalgebras are studied. The indecomposability of baby Kac modules and baby Verma modules is proved in some situation. In particular, for the classical Lie superalgebra of type $A(n|0)$, the baby Verma modules $Z_\chi(\lambda)$ are proved to be simple for any regular nilpotent $p$-character $\chi$ and typical weight $\lambda$. Moreover, we obtain the dimension formulas for projective covers of simple modules with $p$-characters of standard Levi form.

Keywords: restricted Lie superalgebra; $\chi$-reduced representation; indecomposable module; simple module; $p$-character

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1. Introduction

The finite-dimensional simple Lie superalgebras over the field of complex numbers were classified by Kac in the 1970s (cf. [8]). Although until now, the classification of finite-dimensional simple (restricted) Lie superalgebras over a field of prime characteristic has not yet been completed, there has been increasing interest in modular representation theory of restricted Lie superalgebras in recent years. W. Wang and L. Zhao [15], [16] initiated and developed systematically the modular representations of Lie superalgebras over an algebraically closed field of characteristic $p > 2$. In [15], the super version of the celebrated Kac-Weisfeiler Property was shown to hold for the basic classical Lie superalgebras, which by definition admit an even non-degenerate supersymmetric bilinear form and whose even subalgebras are reductive. There also has been increasing interest [1], [2], [3], [4], [9], [11] in modular representation theory of algebraic supergroups in connection with other areas in recent years. Indeed,

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the modular representation theory of supergroups and Lie superalgebras has found remarkable applications to classical mathematics (see [11] for references and some historical remarks). Representations of Cartan type Lie superalgebras of prime characteristic were studied in [12], [13], [14], [17], [18], [19].

The modular representations of restricted Lie algebras of prime characteristic have been developed over the years (see [7] for a review). The corresponding question for restricted Lie superalgebras naturally arises. The present work has been largely motivated by the representation theory of modular Lie algebras (cf. [5], [7]). Let \((g, [p])\) be a finite-dimensional restricted Lie superalgebra over an algebraically closed field \(F\) of characteristic \(p > 2\). It is obvious that for each \(x \in g_0\), the element \(x^p - x^{[p]}\) is even and central in the universal enveloping superalgebra \(U(g)\). Let \(Z\) denote the central subalgebra of \(U(g)\) generated by all the elements \(x^p - x^{[p]}\) with \(x \in g_0\), which is the so-called \(p\)-center. Since each irreducible \(g\)-module is finite-dimensional (cf. [15], [20]), the Lie superalgebra version of Schur’s Lemma [8], §1.1.6, implies that the \(p\)-center \(Z\) acts by scalars on any irreducible \(g\)-module. Then there exists a unique \(\chi \in g^*_0\) such that \(x^p \cdot v - x^{[p]} \cdot v = \chi(x)^p v\), for all \(x \in g_0\), \(v \in M\). Therefore, \(M\) is a module for the finite-dimensional superalgebra \(U_\chi(g) = U(g)/(x^p - x^{[p]} - \chi(x)^p | x \in g_0)\), where \((x^p - x^{[p]} - \chi(x)^p | x \in g_0)\) denotes the ideal of \(U(g)\) generated by all the elements \(x^p - x^{[p]} - \chi(x)^p\) with \(x \in g_0\). The superalgebra \(U_\chi(g)\) is called the \(\chi\)-reduced enveloping superalgebra. More generally, a \(g\)-module \(M\) is said to have a \(p\)-character \(\chi\) provided that \(x^p \cdot v - x^{[p]} \cdot v = \chi(x)^p v\), for all \(x \in g_0, v \in M\), or equivalently, it is a \(U_\chi(g)\)-module.

This paper is structured as follows. In Section 2, we recall some basic notation and properties for restricted Lie superalgebras. Section 3 is devoted to studying representations of restricted Lie superalgebras with an admissible \(\mathbb{Z}\)-grading of depth one. The baby Kac modules are proved to be indecomposable and have simple socle. In the final section, we study representations of the classical Lie superalgebra \(\mathfrak{sl}(n + 1\mid 1)\) with \(p\)-character of standard Levi form. The baby Verma modules are proved to be indecomposable. When \(\chi\) is regular nilpotent and \(\lambda\) is typical, the baby Verma module \(Z_\chi(\lambda)\) is simple. Moreover, we obtain the dimension formulas for projective covers of simple modules with \(p\)-characters of standard Levi form.

2. Preliminaries

Throughout this paper, \(F\) is assumed to be an algebraically closed field of prime characteristic \(p > 2\). All modules (vector spaces) are over \(F\) and finite-dimensional.

The following notion of restricted Lie superalgebras is a generalization of the one for restricted Lie algebras (see [6]).