NONLINEAR CONTROL PROBLEMS

INVERTIBILITY OF SYSTEMS WITH UNSTABLE ZERO DYNAMICS

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We consider the problem of robust inversion of an uncertain dynamical system with unstable zero dynamics. The solution of the problem is reduced to estimating in real time the bounded solution of an unstable linear differential equation. Estimation algorithms are proposed for one- and two-output systems. Convergence of the inversion algorithms is assessed and the effect of observation errors on the algorithms, i.e., their robustness, is investigated.

1. Introduction

Problems of inverse dynamics in various settings are a key issue in modern control theory. Many Western and Russian authors have dealt with this topic [1 – 7]. Inverse dynamic problems are important for a whole range of applications, such as guiding an aircraft along a specified trajectory, identification of dynamic system parameters and noise, etc. We distinguish two classes of inversion problems: numerical problems and real-time inversion problems. Moreover, inversion algorithms are required to be robust, i.e., stable under changes of problem parameters within finite bounds. In this paper, we deal with robust algorithms that invert finite-dimensional linear systems in real time.

The inversion problem is solved by reducing the original problem to the problem of stabilization of an uncertain system by output feedback. This enables us to apply the well-developed apparatus of automatic control theory [7 – 14].

2. Statement of the Problem

We consider the inversion of the dynamical system \( P\xi(t) = w(t) \) with a linear operator \( P \). Observing the scalar output \( w(t) \), we have to estimate the unknown scalar input \( \xi(t) \). We use an approach based on a controlled model \( Pu(t) = \tilde{w}(t) \) of the original system, where the control \( u(t) \) is chosen so that the model output \( \tilde{w}(t) \) is close in some sense to the system output \( w(t) \) (approaches it asymptotically or with prespecified accuracy). The problem is thus reduced to the problem of stabilization of the linear uncertain system \( P(u(t) - \xi(t)) = y(t) = \tilde{w}(t) - w(t) \). If the control \( u(t) \) is such that \( y(t) = 0 \), then \( u(t) = \xi(t) + \delta(t) \), where \( \delta(t) \) is a function from the kernel of the operator \( P \). We thus see that in general (without additional assumptions) the system is invertible up to functions from the kernel of the operator \( P \).

In [8 – 13] this approach has been applied to invert the linear system

\[
\dot{z} = Az + b\xi(t), \quad w(t) = cz, \tag{1}
\]

where \( z(t) \in \mathbb{R}^n \) and \( w(t), \xi(t) \) are scalar functions; \( A, \ b, \) and \( c \) are appropriately dimensioned constant matrices.

Following the proposed approach, we consider the controlled model

\[
\dot{\tilde{z}} = A\tilde{z} + bu(t), \quad \tilde{w}(t) = c\tilde{z}, \tag{2}
\]

and reduce the problem to stabilization of the system in deviations

\[ \dot{x} = Ax + b(u(t) - \xi(t)), \quad y(t) = cx, \tag{3} \]

where \( x = \tilde{z} - z \in \mathbb{R}^n \). Assume that for the system (3) the pair \( \{A, b\} \) is controllable, the pair \( \{A, c\} \) is observable, and the transfer function has the form

\[ W(s) = c(sE - A)^{-1}b = \frac{\beta_m(s)}{\alpha_n(s)}, \tag{4} \]

where \( \beta_m(s) \) and \( \alpha_n(s) \) are polynomials of appropriate degree in \( s \). The polynomial \( \beta_m(s) \) defines the zero dynamics of system (3). The kernel of the operator \( P \) consists of the functions \( \delta(t) \) that satisfy the \( m \)th order differential equation

\[ \beta_m \left( \frac{d}{dt} \right) \delta(t) = 0. \]

In [8–13] it has been assumed that the system has stable zero dynamics (i.e., the polynomial \( \beta_m(s) \) is stable). Then the kernel of the operator \( P \) consists of exponentially decreasing functions.

Our objective is to solve the inversion problem for the case when the polynomial \( \beta_m(s) \) has unstable roots, i.e., the kernel of the operator \( P \) contains also exponentially increasing functions. This assumption substantially complicates the problem.

3. Inversion of Scalar Systems

We apply the following approach to solve the inversion problem. Let the polynomial \( \beta_m(s) \) be representable in the form

\[ \beta_m(s) = \beta_{m_1}(s)\beta_{m_2}(s), \quad m_1 + m_2 = m, \tag{5} \]

where the roots of the polynomial \( \beta_{m_1}(s) \) are in the left halfplane of \( \mathbb{C} \) and the roots of \( \beta_{m_2}(s) \) are in the right halfplane. Without loss of generality we assume that the roots of the polynomial \( \beta_{m_1}(s) \) satisfy the condition \( \Re \gamma_i < -\gamma < 0, \quad i = 1, \ldots, m_1 \), and all the roots of \( \beta_{m_2}(s) \) satisfy the condition

\[ \Re \lambda_i > \mu > 0, \quad i = 1, \ldots, m_2. \tag{6} \]

We rewrite system (3) in operator form using (5):

\[ \frac{\beta_{m_1}(s)\beta_{m_2}(s)}{\alpha_n(s)}\xi = y. \tag{7} \]

Assume that the output signal \( \xi(t) \) is from class \( \Omega^{m_2+1} = \{ \xi(t) : |\xi^i(t)| \leq \xi^i, i = 0, \ldots, m_2 + 1 \} \), i.e., the class of \( (m_2 + 1) \) times differentiable functions whose derivatives up to order \( m_2 + 1 \) are uniformly bounded. Consider a new signal \( \bar{y}(t) \):

\[ \bar{y}(t) = \beta_0\xi(t) + \beta_1\xi'(t) + \ldots + \beta_{m_2}\xi^{(m_2)}(t), \tag{8} \]

where \( \beta_i \) are the coefficients of the polynomial \( \beta_{m_2}(s) \) (without loss of generality we assume in what follows that \( \beta_{m_2} = 1 \)). Since \( \xi \in \Omega^{m_2+1} \), then obviously \( \bar{y} \in \Omega^1 \). Then system (7) in this notation takes the form

\[ \frac{\beta_{m_1}(s)}{\alpha_n(s)} \bar{y} = y, \quad \beta_{m_2}(s)\xi = \bar{y}. \tag{9} \]