STABILIZATION OF HYPEROUTPUT SYSTEMS WITH UNCERTAINTY BY CHOICE OF ZERO DYNAMICS

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We propose an approach to output stabilization of multiply connected control systems with uncertainty based on structural decomposition of the original system and asymptotic invariance methods. The proposed approach solves the stabilization problem for minimum-phase systems. Bounds are obtained on the rate of convergence of the stabilization algorithms. Conditions are derived expanding the class of vector controlled systems with uncertainty that are stabilizable by asymptotic invariance methods.

Introduction

An approach to the synthesis of continuous stabilizing feedback for systems with uncertainty has been developed in [1–3] based on asymptotic invariance ideas. Asymptotic invariance methods make it possible to synthesize continuous feedback and design a control system with given computed characteristics. In [4] asymptotic invariance methods have been generalized for output stabilization of a special class of multiply connected systems with uncertainty. The membership of a system with uncertainty in this class is determined by the decomposition algorithm proposed in [4].

In the present article we apply decomposition algorithms identifying the zero dynamics of systems [5] to expand (compared with [4]) the class of multiply connected systems with uncertainty for which the stabilization problem is solved by asymptotic invariance methods.

1. Statement of the Problem and Notation

We consider a controlled system with uncertainty

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + B(u(t) + \psi(t, x)), \quad x(0) = x_0, \quad u \in U(t, x), \quad \psi \in K(t, x); \\
y(t) &= Cx(t),
\end{align*}
\]

where for every \( t \geq 0 \): \( x(t) \in \mathbb{R}^n \) is the state vector, \( A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}, C \in \mathbb{R}^{p \times n} \) are known constant parameters of system (1); \( U: \mathbb{R}_+ \times \mathbb{R}^n \rightarrow \mathbb{R}^m \) is some many-valued mapping, \( K: \mathbb{R}_+ \times \mathbb{R}^n \rightarrow \mathbb{R}^m \) is a piecewise-continuous in \( t \) and continuous in \( x \) many-valued mapping with linear bounded growth \( |K_i(t, x)| \leq \psi_i^+|x(t)| \); \( \psi_i^+ > 0 \) are known constants \( (i = 1, n) \); \( |\cdot| \) is the Euclidean norm in the space \( \mathbb{R}^n \); the pair \( (A, C) \) is controllable, the pair \( (A, B) \) is controllable, the pair \( (A, C) \) is observable.

For every \( u \in U(t, x) \) and \( \psi \in K(t, x) \) the controlled system (1) is called a closed-loop control system. In what follows the following assumptions hold:

**Assumption A1.** The pair \( (A, B) \) is controllable, the pair \( (C, A) \) is observable, i.e., the system (1) is in general position.

**Assumption A2.** \( \text{rank } B = m, \text{ rank } C = p \), i.e., the matrices \( C \) and \( B \) are of full rank.
Assumption A3. \( m \leq p \leq n \), and rank \( CB = m \), i.e., the matrix \( CB \in \mathbb{R}^{p \times m} \) is of full rank.

Assumption A4. The zero dynamics of system (1) either does not exist or is stable, i.e., the system is minimum-phase.

Recall that the zero-dynamics spectrum of vector system (1) is determined by Rosenbrock’s matrix [13]:

\[
R(s) = \begin{pmatrix}
    sI - A & -B \\
    \vdots & \ddots & \ddots \\
    C & \ddots & \ddots & O
\end{pmatrix} \in \mathbb{C}^{(n+p) \times (n+m)}.
\]

The number \( \lambda \in \mathbb{C} \) is in the zero-dynamics spectrum of system (1) if

\[
\text{rank } R(\lambda) < n + m
\]

(by Assumption A3, \( n + m \leq n + p \)).

Using Assumption A3, we consider the following cases:

Case 1. \( m = p \).

In this case the zero-dynamics characteristic polynomial has the form

\[
\beta(s) = \det R(s).
\]

Since the matrix \( CB \) is of full rank, \( \deg \beta(s) = n - p \), and system (1) can be reduced by some nonsingular transformation [14] to the form

\[
\begin{aligned}
    \dot{x}' &= A_{11} x' + A_{12} y, \quad x' \in \mathbb{R}^{(n-p)}, \\
    \dot{y} &= A_{21} x' + A_{22} y + CB(u + \psi),
\end{aligned}
\]

(2)

where \( \det(sI - A_{11}) = \beta(s) \) is a Hurwitz polynomial by Assumption A4. In this case, the spectrum \( \beta(s) \) is entirely determined by the parameters of system (1).

Case 2. \( p > m \).

In this case, the number of outputs exceeds the number of inputs and the zero-dynamics characteristic polynomial is determined in the following way. Consider all possible \( m \)-dimensional outputs of system (1):

\[
y_{i_1,\ldots,i_m} = (y_{i_1}, \ldots, y_{i_m}).
\]

In total there are \( C_p^m \) such outputs. For each of them we can define a zero-dynamics characteristic polynomial by the output \( y_{i_1,\ldots,i_m} \):

\[
\beta_{i_1,\ldots,i_m}(s) = \det \begin{pmatrix}
    sI - A & -B \\
    \vdots & \ddots & \ddots \\
    C_{i_1,\ldots,i_m} & \ddots & \ddots & O
\end{pmatrix},
\]