GENERALIZED MAGNETO-THERMOELASTICITY WITH MODIFIED OHM'S LAW UNDER THREE THEORIES

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The present paper is concerned with an electro-magneto-thermoelastic coupled problem for a homogeneous, isotropic, thermally and electrically conducting two-dimensional half-space solid whose surface is subjected to a thermal shock. The modified Ohm's law, including the temperature gradient and charge density effect, to the equations of the theory of generalized electromagneto-thermo-elasticity under the coupled (CD), Lord-Shulman (LS), and Green-Lindsay (GL) model of generalized thermoelasticity, has been introduced. An initial magnetic field acts parallel to the plane boundary of the half-space. The normal mode analysis together with eigenvalue approach techniques are used to solve the resulting nondimensional coupled equations for the three theories. Numerical results for the temperature, displacements, thermal stress, and induced magnetic field distributions are presented graphically for three cases and discussed.

Keywords: Magneto-thermoelasticity, modified Ohm’s Law, relaxation time, normal mode analysis, eigenvalue approach.

1. Introduction

Biot [3] formulated the CD theory to eliminate the paradox inherent in the classical uncoupled thermoelasticity theory that “elastic changes have no effect on temperature.” In CD theory, the equation of motion is of hyperbolic type, but the heat equation is of diffusion type, predicting an infinite speed for thermal signals contrary to the physical observations. Predications based on the parabolic temperature equation can become measurably false in the very low temperature region [4].

Generalized thermoelasticity theories have been developed with the objective of removing the paradox of infinite speeds for thermal signals inherent in CD theory. In 1967, Lord and Shulman [5] proposed a model, referred to as the LS model. Their goal was to eliminate the paradox of infinite velocity of thermoelastic disturbances. In the LS model, the heat conduction equation is of hyperbolic type and is closely connected with the theories of second sound.

After few years, Green and Lindsay [6] formulated a more explicit model of generalized thermoelasticity (the GL model) with two relaxation time parameters using the entropy production inequality of Green and Laws [7], and they modify both the energy equation and constitutive equations. The GL model also admits second sound without violating the classical Fourier’s law. This model is also called temperature-rate-dependent thermoelasticity (TRDTE).

The study of electro-magneto-thermoelastic interactions which deals with the interactions among strain, temperature, and electromagnetic fields in a thermoelastic material, is of great practical importance due to its extensive uses in diverse fields, such as geophysics (for understanding the effect of the Earth’s magnetic field on seismic waves), damping of acoustic waves in a magnetic field, designing machine elements like heat exchangers, boiler tubes where temperature-induced elastic deformation occurs, biomedical engineering (problems involving thermal stress), emissions of electromagnetic radiations from nuclear devices, development of a highly sensitive superconducting magnetometer, electrical power engineering, plasma physics, etc. [1, 2].

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Owing to the mathematical difficulties encountered in two-dimensional multi-field coupled generalized heat conduction problems, the problems become too complicated to obtain analytical solution. Instead of analytical methods, several authors applied numerical techniques such as the finite difference method, finite element method, boundary value method, etc. for solving such kinds of problems. Based on the theories of generalized thermoelasticity, Ezzat et al. [21, 22] applied normal mode analysis to solve some two-dimensional electro-magneto-thermoelastic problems. He and Li [23] and Allam et al. [24] also solved electro-magneto-thermoelastic problems using normal mode analysis. Using the normal mode analysis technique, we actually get the solutions in the Fourier-transformed domain. To apply the normal mode analysis, we have to assume that all the relations are sufficiently smooth on the real axis such that the normal mode analysis of all the functions exist.

In this paper, an electro-magneto-thermoelastic coupled two-dimensional problem with modified Ohm’s law for a homogeneous, isotropic, thermally and electrically conducting half-space solid subjected to a thermal shock on its surface is considered based on the CTE, LS, and GL models. The generalized electro-magneto-thermoelastic coupled equations are first formulated. Exact expressions for the considered physical quantities are obtained by applying the normal mode analysis and eigenvalue approach [25–27]. The distributions of the considered physical quantities are represented graphically and the results in three different cases are discussed. From the distributions, it can be noticed that a wave type heat propagates in the medium with finite speeds.

2. Governing Equations and Formulation of the Problem

We consider a homogeneous, isotropic, thermally and electrically conducting thermoelastic half-space under the CTE, LS, and GL theories of thermoelasticity in two-dimensional space that fills the region

\[ \Omega = \{(x, y, z): 0 \leq x < \infty, \, -\infty < y < \infty, \, -\infty < z < \infty\} \]

subjected to a thermal shock on the bounding plane to the surface \( x = 0 \). A magnetic field with constant intensity, namely \( \mathbf{H} = (0, 0, H_0) \), acts parallel to the bounding plane (taken as the direction of the \( z \)-axis). The surface of the half-space is subjected initially \( (t = 0) \) to a thermal shock that is a function of \( y \) and \( t \). Thus, all the quantities considered in this problem will be functions of the time variable \( t \) and coordinates \( x, y \).