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Abstract. In this paper, we propose a rigorous dynamic supernetwork theory for the integration of social networks with financial networks with intermediation in the presence of electronic transactions. We consider decision-makers with sources of funds, financial intermediaries, as well as demand markets for the various financial products. Through a multilevel supernetwork framework consisting of the financial network and the social network we model the multicriteria decision-making behavior of the various decision-makers, which includes the maximization of net return, the maximization of relationship values, and the minimization of risk. Increasing relationship levels in our framework are assumed to reduce transaction costs as well as risk and to have some additional value for the decision-makers. We explore the dynamic evolution of the financial flows, the associated product prices, as well as the relationship levels on the supernetwork until an equilibrium pattern is achieved. We provide some qualitative properties of the dynamic trajectories, under suitable assumptions, and propose a discrete-time algorithm which is then applied to track the evolution of the relationship levels over time as well as the financial flows and prices. The equilibrium pattern yields, as a byproduct, the emergent structure of the social and financial networks since it identifies not only which pairs of nodes will have flows but also the size of the flows, i.e., the relationship levels and the financial transactions.

Key words: social networks, financial networks, variational inequalities, projected dynamical systems, supernetworks

1. Introduction

The literature on social networks is considerable and growing (cf. Freeman, 2000; Krackhardt, 2000) but it has developed largely in the discipline of sociology without much emphasis on the economic aspects of human decision-making including those
surrounding financial transactions. The literature on financial networks, in turn, as noted in Nagurney and Siokos (1997), dates to the classical work of Quesnay (1758) who depicted the circular flow of financial funds in an economy as a graph. It has yielded a wealth of models, supported by a variety of methodological tools (cf. Nagurney (2003) and the references therein), that aim to not only capture the economic behavior of the various decision-makers involved in the allocation and uses of funds but also allow for the computation of the optimal/equilibrium financial flows and prices.

In this paper, our goal is to provide a theory for the integration of social networks with financial networks that explores the evolution of financial transactions over time coupled with the evolution of relationships between the financial decision-makers; that is, those with sources of funds, the intermediaries, as well as the consumers of the various financial products. In our model, the decision-makers can choose between the possibility of physical as well as electronic transactions along the lines proposed by Nagurney and Ke (2003). In addition, they can, within a multicriteria decision-making context, also select their relationship levels which, in turn, affect the transaction costs and the risk and are assumed to hold some additional value. Establishing (and maintaining) the relationship levels, however, incurs some costs that need to be borne by the decision-makers. Hence, our framework, unlike much of the literature in social networks, considers flows in the form of relationship levels between tiers of decision-makers.

The role that relationships play in financial networks has gained increasing attention and has been studied analytically as well as empirically in several different contexts. Ghatak (2002) as well as Anthony (1997) described the role of social networks in the context of micro-financing. Boot and Thakor (2000), in turn, developed a model to determine the effect of an increase of interbank competition and capital market competition on relationship lending. Sharpe (1990), Petersen and Rajan (1994), Berger and Udell (1995) as well as Petersen and Rajan (1995) dealt with the connection between relationships and lending. Petersen and Rajan (1994) and Arrow (1998) concluded that bank-firm ties are more critical to lending markets than classical theory suggests, and that social relationships and networks affect who gets capital and at what cost. Wilner’s (2000) findings concerning the optimal pricing, lending, and renegotiation strategies for companies with relationships such that one company depends strongly on another one are compatible with those in the relationship-lending literature. Jackson (2003) surveyed the recent literature on network formation with a focus on game theoretic models as well as networks of relationships and applications. He also emphasized that networks of relationships play an important role in many economic situations.

Uzzi (1997) and DiMaggio and Louch (1998), basing their research on the sociological research on lending, suggested that social relationships and networks affect personal and corporate financial dealings. Uzzi (1999) pointed out that firms are more likely to get loans and to receive lower interest rates on loans if social relationships and network ties exist. Burt (2000) along with cited references overviews