What Drives Short Rate Dynamics? A Functional Gradient Descent Approach

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Abstract  Functional gradient descent (FGD), a recent technique coming from computational statistics, is applied to the estimation of the conditional moments of the short rate process with the goal of finding the main drivers of the drift and volatility dynamics. FGD can improve the accuracy of some reasonable starting estimates obtained using classical short rate models introduced in the literature. It exploits the predictive information of an enlarged set of variables, including yields at other maturities, time, and macroeconomic indicators. Fitting this methodology to the time series of monthly US 3-month Treasury bill rates, we find that the drift dynamics react mostly in a non-linear way to changes in macroeconomic variables, whereas volatility dynamics are subjected to time-dependent regime-switches. Finally we show the superior performance of the final predictions obtained by applying FGD in a forecasting exercise.

Keywords  Functional gradient descent · Short rate process · Macroeconomic variables · Time-varying drift and volatility dynamics

JEL Classification  C14 · C52 · E43 · E44 · E47

1 Introduction

In the last 20 years the academic community has devoted a lot of attention to the understanding, modeling, and forecasting of the short-term interest rate process, giving rise to an enormous number of studies published in the literature. The main reason for such strong interest is the key role played by the short rate in many practical applications such as the pricing of bond and interest-rate dependent derivative
securities and (interest rate) risk management. In fact, the short rate process is generally thought to be the most important state variable driving the whole term structure of interest rates.

Recent studies in continuous time have focused on the description and estimation of (possible) non-linearities in the drift and diffusion functions of the short rate using parametric and semi- or even non-parametric approaches. Among others, it is worthwhile citing the results of Aït-Sahalia (1996b) who proposed a general parametric specification nesting most existing parametric interest rate models, in particular the classical Vasicek (1977) and Cox et al. (1985) models. Given the potential risk of model misspecification inherent in the parametric assumption yielding significant economic implications in the pricing of interest rate derivatives, nonparametric modeling has received considerable attention. Aït-Sahalia (1996a) proposed a non-parametric estimator of the diffusion function from discretely observed data in connection with a parametric drift function. Stanton (1997) proposed nonparametric estimators of the drift and diffusion functions based on different orders of approximation of the Itô process using the infinitesimal generator and Taylor series expansions. Bandi and Phillips (2003) generalized the nonparametric approach to recurrent diffusion processes, relaxing the assumption of stationarity for the short rate process. Sam and Jiang (2009) extended the nonparametric estimator proposed by Stanton (1997) by incorporating the informative potential of a panel of yields in the nonparametric estimation. They found in simulations that the proposed estimator can lead to significant efficiency gains relative to the nonparametric estimator constructed using only the time series of observed short rates, reducing problems related to spurious non-linearities in the drift function (see, for example, Pritsker 1998, or Chapman and Pearson 2000).

In the discrete setting, different time series models of the short rate dynamics have recently been proposed in order to take into account the regime-switching behavior and the heteroskedasticity and high persistence over time of the short rate process. These models also try to relate short rates with macroeconomic fundamentals like indicators of inflation and real activity. In their studies, Gray (1996), Bansal and Zhou (2002), Bansal et al. (2004), Audrino (2006), Audrino and De Giorgi (2007), and Audrino and Medeiros (2011) showed empirically that the incorporation of regime shifts (of a Markovian or threshold type) in the time series model for the short rate process significantly improves the accuracy of the estimates and predictions of short rate conditional means and variances, with important implications for the pricing of interest-rate sensitive derivative instruments. Moreover, motivated by the results illustrated in Ang and Piazzesi (2003), Diebold et al. (2006), Ang et al. (2007), and Rudebusch and Wu (2008), who considered macroeconomic fundamentals as observable factors in yield curve modeling, Audrino (2006) and Audrino and Medeiros (2011) included macroeconomic information as predictors and regime-switching variables in the construction of the short rate model, showing that such information is highly relevant for improving the fit and the prediction of the models. Indices of inflation and real activity are found to be among the main determinants, driving both the local short rate conditional dynamics in the different regimes and the way in which regime switches are determined.

This study follows the idea of extending the information set for the estimation of the first two conditional short rate moments (i.e. drift and diffusion functions) by