Axisymmetric flow of two fluids in a pulsating pipe

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Abstract The motion of a viscous thread surrounded by an annular viscous layer inside a pulsating cylindrical pipe whose radius is a periodic function of time is investigated. At zero Reynolds number, a stagnation-point-type solution may be written down in closed form. A Floquet linear stability analysis for Stokes flow reveals the pulsations either decrease or increase the growth rate of longwave disturbances depending on the initial radius of the thread. For a moderate-sized initial thread radius, increasing the amplitude of the pulsations decreases the critical wavenumber for instability to below the classical Rayleigh threshold. Increasing the viscosity contrast, so that the fluid in the annular layer becomes more viscous than the fluid in the thread, tends to decrease the growth rate of disturbances. In the second part of the paper, the basic stagnation-point-type flow at arbitrary Reynolds number is computed using a numerical method on the assumption that the interface is a circular cylinder at all times. During the motion, either the thread radius tends to increase and the thickness of the annular layer decreases, or else the thread tends to thin and the thickness of the annular layer increases, depending upon the initial conditions and the parameter values. For a judicious choice of initial condition, a time-periodic exact solution of the Navier–Stokes equations is identified.

Keywords Interfacial flow · Navier–Stokes flow · Stokes flow

1 Introduction

Pulsating flows involving two or more fluid constituents arise in a number of biological applications. In the human digestive tract, liquid and solid meals arriving in the fundus, the upper part of the stomach, are slowly mixed with enzymes through a gentle pulsing of the stomach wall (e.g. [1]). The enzymes can be treated as a layer of viscous fluid which, upon secretion, coat the inside of the gastric wall and surround the core fluid meal before convecting and diffusing into the interior. In healthy human lungs, the thin layer of liquid mucus coating the inside of the airways decreases in thickness during expiration (e.g. [2]). During normal rhythmic breathing, closure of the smaller airways may occur due to the formation of a liquid mucus lens, or through the closure of the compliant elastic airway wall, or through a combination of these two effects. Monitoring the occurrence of closure is one means of determining healthy lung function (e.g. [3]).
Motivated by examples such as these, in the present paper we undertake a general theoretical investigation of the effect of wall pulsations on an axisymmetric arrangement of two fluids in a cylindrical pipe.

Lord Rayleigh [4, 5] investigated the stability of a quiescent inviscid or viscous liquid thread suspended in vacuum. When the wavelength of an axisymmetric perturbation is larger than the circumference, the thread is unstable and disintegrates into a file of liquid drops (e.g. [6]). The more general scenario with a viscous thread at rest inside an ambient viscous fluid was studied by Weber [7] and Tomotika [8], who identified the same critical wavelength for instability and calculated the wavelength of the most rapidly growing disturbance. Goren [9] discussed the stability of a quiescent liquid film coating a straight wire or wetting the inside of a hollow cylindrical tube, and computed the wavelength of the most dangerous disturbance. Hammond [10] studied the nonlinear evolution of a thin annular film, which surrounds a cylindrical viscous thread and coats the inside of a circular cylinder, and showed that the film is unstable to longwave disturbances which break up the fluid film into disconnected lobes. More recently, Kwak and Pozrikidis [11] extended Tomotika’s analysis to allow for the presence of confining walls and, in particular, to include the effect of an insoluble surfactant at the interface between the fluids.

Tomotika [12] examined the stability of a viscous thread undergoing extensional flow inside a surrounding viscous fluid. His analysis was improved upon by Mikami et al. [13], who also compared their results with experiments. The intriguing feature of this problem is the competition between the stabilising action of the extensional flow, which tends to oppose the growth of interfacial disturbances, and the capillary instability which comes into play as the wavelength of a disturbance is stretched by the flow beyond the critical Rayleigh limit. Mikami et al. [13] showed that under most conditions an initial disturbance may go through a transitory period of growth due to the capillary instability, but ultimately it decays and the thread is stabilised. This observation was confirmed by Kwak et al. [14], who also accounted for the presence of an insoluble surfactant and compared the results of the linear theory at zero Reynolds number with full boundary-integral simulations of the equations of Stokes flow. Khakhar and Ottino [15] considered the evolution of a viscous thread in an arbitrary linear flow and found results which are in broad agreement with the results for axisymmetric flow.

As noted above, the convective effect of an extensional flow is to lower the amplitude of small disturbances while stretching their wavelength into the regime where they fall prey to the Rayleigh instability. Presumably, if the direction of flow is reversed, convection will tend to amplify linear disturbances but at the same time shorten their wavelength and reduce the chance of the Rayleigh instability. This raises the interesting question of how the thread behaves if the flow is oscillating in time, so that the flow direction reverses every half period. The stability of multi-component axisymmetric oscillating fluid systems has been discussed by a number of authors. For example, Halpern and Grotberg [16] studied the case of a thin viscous film on the inside of a cylindrical pipe surrounding a core fluid undergoing oscillating flow. Blyth and Pozrikidis [17] examined the stability of a pulsating gas column inside either an inviscid or a viscous fluid at zero Reynolds number using Floquet analysis. Of particular relevance to the present work is the two-dimensional study by Blyth [18], which considered the effect of pulsations on the stability of two-layer flow in a channel. Blyth showed that the pulsations tend to raise the growth rate of normal modes, but are unable to destabilise an otherwise stable configuration.

In this paper, we investigate the behaviour of a viscous thread surrounded by an annular viscous layer inside a pulsating cylinder whose radius varies periodically in time. The axisymmetric flow is assumed to adopt a stagnation-point-type structure where the axial component of velocity is proportional to distance along the axis, and the radial component of velocity is independent of the axial coordinate. In the basic configuration, the motion of the wall induces an alternately extensional and contracting flow for which the interface between the two fluids is a circular cylinder at all times. Under these stipulations, an exact solution valid at zero Reynolds number may be written down in closed form. Since the basic flow is periodic in time, we can use Floquet theory to study the development of a linear interfacial disturbance, and compute the growth rates of normal modes. In the second part of the paper, we study the motion of the thread at arbitrary Reynolds number. The flow is again assumed to adopt a stagnation-point-type structure, with a linear dependence on the axial coordinate. Working on the assumption that the interface remains a circular cylinder throughout the motion, we use a numerical method to follow the development of the flow from a specified initial condition and categorise the long-term thread behaviour for different values of the wall amplitude, the viscosity ratio, and the Reynolds number.