Marangoni-induced deformation of evaporating liquid films on composite substrates

Tatiana Gambaryan-Roisman

Abstract Marangoni convection plays an important role in hydrodynamics of evaporating liquid films and sessile drops. Evaporation of liquid films induces unsteady nonuniform temperature distribution across the liquid layer and in a substrate. If the substrate is composed of parts with different thermal properties, the interface temperature distribution becomes non-uniform, leading to appearance of Marangoni stresses, convective vortices, and film deformation. In this article, a model describing evaporation, Marangoni effect and interface dynamics of liquid films on composite substrates is developed. The film dynamics is described in the framework of long-wave theory. The unsteady heat conduction in the substrate is described using the Laplace transform method for semi-infinite substrates and using the separation of variables technique for substrates of finite thickness. The non-uniformity of substrate thermal properties has a pronounced effect on film dynamics.

Keywords Composite materials · Evaporation · Heat conduction · Long-wave theory · Marangoni effect · Thin films

1 Introduction

Evaporation of liquid films and drops is a key phenomenon in many modern technological processes, including ink-jet printing [1], drying of paint films [2, 3], and self-assembly of nanoparticles and porous layers [4–6]. The dynamics of evaporating films and drops is governed by various physical mechanisms, including surface tension, gravity, wettability, and phase change. The Marangoni effect, which arises from the surface tension non-uniformity, in many cases dominates the hydrodynamics of evaporating layers [5, 7–9]. Marangoni effect may lead to hydrodynamic instabilities which are responsible for appearance of vortices in the liquid layer and for spontaneous film rupture. In pure liquids, the surface tension gradients appear due to interface temperature gradients, which lead to thermocapillary convection [10].

Thermocapillary convection and film evolution on heated substrates in the presence or in the absence of evaporation can be controlled by modification of the boundary conditions at the solid–liquid interface. In particular,
the application of spatially non-uniform heating of the substrate leads to permanent deformation of the liquid–gas interface [11,12]. The non-uniform wall heating leads to the temperature non-uniformity at the liquid–gas interface. Since for most of the common liquids the surface tension decreases with increase of temperature, surface tension at the hotter locations of the liquid–gas interface is lower than that at the colder locations. The surface tension gradient results in shear stresses. Under the action of these stresses the liquid in the vicinity of the liquid–gas interface flows from the hotter sites to colder interface sites, which leads to interface deformation.

Thermocapillary convection and film deformation on a heated substrate can be influenced by using substrates with topography [13–17]. If the substrate temperature is higher than that of the environment, the sites on the liquid–gas interface over the topography crests are hotter than the interface sites corresponding to the topography troughs. The temperature non-uniformity along the liquid–gas interface leads to development of thermocapillary convection and to accumulation of the liquid in the grooves or dimples. In the study of Alexeev et al. [14] the Marangoni convection in a liquid film covering a substrate with straight grooves has been studied experimentally and numerically. It has been observed that in a wide range of wall temperatures the Marangoni stresses induce stable two-dimensional vortices.

It has been recently shown that the thermal conductivity of the substrate has a pronounced effect on the flow field inside an evaporating sessile droplet [7,9,18]. Depending on the ratio between the thermal conductivities of the liquid and of the substrate, the liquid flow in the vicinity of the liquid–gas interface can be directed toward the droplet edge or in the opposite direction. It can be expected that the non-uniformity of the substrate thermal conductivity may affect the hydrodynamics of non-isothermal films and droplets. The non-uniformity of the thermal properties may occur in functionally graded materials and in composite materials. The first model describing the effect of non-uniform substrate thermal conductivity on Marangoni convection and film deformation has been recently developed by Gambaryan-Roisman [22]. In this study, the evolution of a liquid film covering a non-uniform substrate has been modeled. It has been assumed that the back side of the substrate was kept at a constant uniform temperature exceeding the temperature of the ambient gas. If the substrate thermal conductivity is non-uniform, the combined thermal resistance of the substrate and the liquid layer varies along the substrate. As a result, the temperature of the liquid–gas interface above the sites of high substrate thermal conductivity is higher than that over the sites with low substrate thermal conductivity. The film deforms in such a way that local interface elevations appear over the sites of low substrate thermal conductivity, whereas local interface depressions develop over the sites of high substrate thermal conductivity.

In many practical situations the films and droplets evaporate from unheated substrates [5]. In this case, the temperature non-uniformity at the liquid–gas interface develops as a result of evaporation. The liquid–gas interface is cooled due to evaporation, and the local temperature drop induces the heat flow from the substrate to the film. If the heat transfer between the back side of the substrate and the environment is slow, the heat transfer in the system substrate-film is essentially unsteady, and the temperatures of the liquid and of the solid decrease monotonically. The rate of the temperature decrease depends on the substrate thermal properties. If the substrate is composite, the non-uniformity of the thermal properties induces temperature gradients at the liquid–gas interface and the Marangoni stresses. This phenomenon has been never studied before.

This article is aimed at the development of a model describing evaporation, Marangoni effect and interface dynamics of a liquid film covering a composite substrate. The film dynamics is described in the framework of a long-wave theory. The unsteady heat conduction in the substrate is solved using the Laplace transform method for semi-infinite substrates and using the separation of variables technique for substrates of finite thickness.

2 Model of heat transfer and film evolution

Consider a thin volatile liquid film of thickness \( h(x,t) \) covering a thick composite substrate of a thickness \( l \) (see Fig. 1). For simplicity, two-dimensional geometry is considered. The gravity acts in the positive \( z \)-direction. Generally, the thermal conductivity and thermal diffusivity of the solid substrate, \( k_s \) and \( \alpha_s \), respectively, may depend on \( x \) and \( z \). We are specifically interested in studying the effect of the spatially periodic variation of thermal