Constant Rebalanced Portfolio Optimization Under Nonlinear Transaction Costs

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Abstract We study the constant rebalancing strategy for multi-period portfolio optimization via conditional value-at-risk (CVaR) when there are nonlinear transaction costs. This problem is difficult to solve because of its nonconvexity. The nonlinear transaction costs and CVaR constraints make things worse; state-of-the-art nonlinear programming (NLP) solvers have trouble in reaching even locally optimal solutions. As a practical solution, we develop a local search algorithm in which linear approximation problems and nonlinear equations are iteratively solved. Computational results are presented, showing that the algorithm attains a good solution in a practical time. It is better than the revised version of an existing global optimization. We also assess the performance of the constant rebalancing strategy in comparison with the buy-and-hold strategy.

Keywords Multi-period portfolio optimization · Constant rebalancing · Transaction cost · Conditional value-at-risk · Market impact cost

1 Introduction

Optimization methods for portfolio selection have been actively studied since the seminal work of Markowitz (1952), and they currently play an important role in financial decision making (see, e.g., Cornuejols and Tüttüncü 2007). The importance of the
multi-period model for long-term portfolio management was recognized early e.g., on Markowitz (1991), and it was first framed as a stochastic control problem (e.g., Merton 1969; Samuelson 1969; Merton 1971) (see Infanger 2006 for detailed references). Closed-form solutions to these problems require very strong assumptions and cannot be generalized in the presence of market frictions such as transaction costs. Moreover, the heavy computational cost of their numerical implementations (see e.g., Brennan et al. 1997; Brandt 1999) have led to alternative stochastic programming models e.g., in Kusy and Ziemba (1986), Mulvey and Vladimirou (1989), Dantzig and Infanger (1993), and Cariño et al. (1994). The most studied model of these alternatives, e.g., in Mulvey and Ziemba (1995), and Ziemba and Mulvey (1998), employs a scenario tree structure for representing the uncertainty of asset values. However, as pointed out by Ermoliev and Wets (1988), the size of the optimization problem grows exponentially with the discretization accuracy. The simulated path model, in which scenarios are represented by sample paths generated by a Monte Carlo simulation, yields a better accurate description of uncertainty (Hibiki 2003). Combining the advantages of the two approaches, Hibiki (2003) proposed a hybrid (bundling simulated path) model that not only describes the uncertainty on a simulated path structure but also enables one to make conditional decisions in a tree structure. In this paper, we shall consider a simulated path model that is compatible with the constant rebalancing strategy.

Among the investment strategies for long-term asset management, the two extremal policies are buy-and-hold and periodical rebalancing (see Perold and Sharpe 1988 for a detailed discussion of the various investment strategies). Constant rebalancing (in other words, fixed mix, constant proportional portfolio and the like) is the most popular. It requires purchases and sales of assets at the beginning of each period in such a way that the investment proportions are restored to the original ones. The advantages of constant rebalancing are as follows: First, we can do without individual decision variables regarding the investment proportion for each period. Second, constant rebalancing is a contrarian investment strategy in which one purchases assets with a declining price and to sells ones with a rising price, and it is supposed to be effective for mid/long term investments. It is known that constant rebalancing achieves the optimal growth rate of wealth if the asset prices in each period are independent and identically distributed (see e.g., Algoet and Cover 1988). Additionally, because of its simplicity, it is easy for financial institutions to explain this rebalancing policy to their customers.

Multi-period portfolio optimization with a constant rebalancing strategy is relatively easy to perform in the case of log-optimal portfolio (see e.g., Cover 1984), in which case the asymptotically optimal portfolio is determined by maximizing the expected log return. However, it becomes a nonconvex problem for which it is difficult to attain a globally optimal solution when a risk measure (e.g., variance of returns) is introduced (Maranas et al. 1997). Even state-of-the-art nonlinear programming (NLP) solvers have difficulty finding locally optimal solutions when the problem size is large. Maranas et al. (1997) studied multi-period mean-variance portfolio optimization with a constant rebalancing strategy for long-term financial planning. They devised a rectangular branch-and-bound algorithm for solving this problem globally. By exploiting the fact that the number of assets is at most nine, their deterministic algorithm attains a globally optimal solution in a practical time. However, they do not consider transaction