On Determining the Percolation Reynolds Number and the Characteristic Linear Dimension for Ideal and Fictitious Porous Media

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Abstract — Various versions of representations of the percolation Reynolds number for porous media with isotropic and anisotropic flow properties are considered. The formulas are derived and the variants are analyzed with reference to model porous media with a periodic microstructure formed by systems of capillaries and packings consisting of spheres of constant diameter (ideal and fictitious porous media, respectively). A generalization of the Kozeny formula is given for determining the capillary diameter in an ideal porous medium equivalent to a fictitious medium with respect to permeability and porosity and it is shown that the capillary diameter is nonuniquely determined. Relations for recalculating values of the Reynolds number determined by means of formulas proposed earlier are given and it is shown that taking the microstructure of porous media into account, as proposed in [1, 2], makes it possible to explain the large scatter of the numerical values of the Reynolds number in processing the experimental data.

Keywords: Reynolds number, characteristic linear dimension, ideal porous medium, fictitious soil, porosity, surface porosity, permeability.

Many experimental data [3–7] show that the basic equation of the theory of flow through porous media, namely, the linear Darcy’s law, has an upper limit of applicability. By analogy with pipeline hydromechanics, as a criterion of applicability of Darcy’s law, the critical value of the percolation Reynolds number has been used, especially because initially the deviation from Darcy’s law was related with laminar-turbulent flow transition [4]. Subsequently, it was established that the deviation from Darcy’s law is associated with the effect of the inertia forces [3].

In order to write the expression for the Reynolds number it is necessary to determine the characteristic linear dimension of the porous medium and go over from the “true” average flow velocity $v$, which is hard to determine for real porous media, to the filter velocity $w$. Experimental data for artificial bulk (fictitious) porous media were used in deriving the first relations proposed for determining the Reynolds number and the effective diameter of the particles (sphere) was taken as the characteristic linear dimension [8]. However, such a determination of the characteristic dimension for real and ideal porous media is meaningless because of the fundamentally different geometry and structure of the void space. Therefore, the characteristic linear dimension for porous oil and gas reservoirs was subsequently also determined both using dimension theory and as the effective capillary diameter in the rigid tube model [9, 10], and, in addition to these representations, for sintered metal materials other considerations were also used [11, 12]. However, all the formulas proposed relate to isotropic media and neglect the features of the structure of the void space. Moreover, different formulas give significantly different numerical values of the Reynolds number which often cannot be recalculated without establishing additional equivalence relations. For different types of porous media the same formula can give values differing by two orders of magnitude. Therefore, we will analyze the determination of the percolation Reynolds number for the most frequently used models of ideal and fictitious porous media for which the relations between the percolation-capacity parameters were established in [5, 13–15].
1. PERCOLATION REYNOLDS NUMBER FOR AN IDEAL POROUS MEDIUM

We will consider the model of an ideal porous medium in the form of a periodic lattice composed of three mutually perpendicular systems of circular tubes (capillaries). To each system of tubes we will assign its own linear dimension (diameter) $d_\alpha$ and a packing period $a_\alpha$, $\alpha = 1, 2, 3$. The presence of a periodic structure with fixed dimensions in the ideal model makes it readily possible to calculate the geometric and hydrodynamic characteristics of the porous medium [1]. For the ideal model the porosity $m$, the specific pore surface $\Sigma$, the surface porosity $s_\alpha$, and the permeability $k_\alpha$ are respectively equal to

$$ m = \frac{\pi d_i^2}{4a_1a_2a_3}, \quad \Sigma = \frac{\pi d_i^2}{a_1a_2a_3}, \quad s_\alpha = \frac{\pi d_i^2}{4a_\beta a_\gamma}, \quad k_\alpha = \frac{\pi d_i^4}{128a_\beta a_\gamma} $$

(1.1)

Here and in what follows, as a rule, the Greek subscripts denote the number of the tube system and form a cyclic permutation of the numbers 1, 2, 3; the summation convention applies to the recurring Latin subscripts, but there is no summation over the Greek subscripts.

For a single capillary in the $\alpha$-th capillary system the Reynolds number is defined by the expression

$$ Re_\alpha = \frac{v_\alpha d_\alpha \rho}{\mu} $$

(1.2)

where $v_\alpha$ is the average capillary velocity (true filter velocity), and $\rho$ and $\mu$ are the fluid density and viscosity, respectively.

In order to go over from the hydraulic Reynolds number (1.2) to the percolation Reynolds number it is necessary to replace $v_\alpha$ by $w_\alpha$ and express the characteristic linear dimension $d_\alpha$ in terms of the geometric and percolation characteristics. The relation between $v_\alpha$ and $w_\alpha$ is given [2] by

$$ w_\alpha = s_\alpha v_\alpha = \frac{m}{\phi_\alpha} v_\alpha, \quad \phi_\alpha = \frac{m}{s_\alpha} $$

(1.3)

where $\phi_\alpha$ is the structure coefficient for the $\alpha$-th capillary system.

In determining $d_\alpha$, we can use various combinations of relations (1.1). Usually, the following representation in terms of the permeability and porosity is employed:

$$ k_\alpha = \frac{\pi d_i^4}{128a_\beta a_\gamma} = \frac{d_i^2}{32} s_\alpha = \frac{d_i^2}{32} \frac{m}{\phi_\alpha} $$

(1.4)

hence

$$ d_\alpha = \sqrt{\frac{32 \phi_\alpha k_\alpha}{m}} $$

(1.5)

However, other relations can also be used in order to represent the capillary diameter in terms of the percolation-capacity properties. In particular, we can use the generalized Kozeny-Carman formula [2]

$$ k_\alpha = \frac{m^3}{(f_\alpha \phi_\alpha \Sigma^2)} $$

(1.6)

where $f_\alpha$ is the shape factor for the $\alpha$-th capillary system, which can be determined from the equality [2]

$$ \frac{m^2}{f_\alpha \Sigma^2} = \frac{d_i^2}{32} $$

(1.7)

whence

$$ d_\alpha = \sqrt{\frac{32 m}{f_\alpha \Sigma}} $$

(1.8)