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Ten Misconceptions from the History of Analysis and Their Debunking

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Abstract The widespread idea that infinitesimals were "eliminated" by the "great triumvirate" of Cantor, Dedekind, and Weierstrass is refuted by an uninterrupted chain of work on infinitesimal-enriched number systems. The elimination claim is an oversimplification created by triumvirate followers, who tend to view the history of analysis as a pre-ordained march toward the radiant future of Weierstrassian epsilontics. In the present text, we document distortions of the history of analysis stemming from the triumvirate ideology of ontological minimalism, which identified the continuum with a single number system. Such anachronistic distortions characterize the received interpretation of Stevin, Leibniz, d'Alembert, Cauchy, and others.

Keywords Abraham Robinson · Adequality · Archimedean continuum · Bernoullian continuum · Cantor · Cauchy · Cognitive bias · Completeness · Constructivism · Continuity · Continuum · du Bois-Reymond · Epsilontics · Felix Klein · Fermat-Robinson standard part · Infinitesimal · Leibniz–Łoś transfer principle · Limit · Mathematical rigor · Nominalism · Non-Archimedean · Simon Stevin · Stolz · Sum theorem · Weierstrass

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1 Introduction

Here are some common claims. The founders of infinitesimal calculus were working in a vacuum caused by an absence of a satisfactory number system. The incoherence of infinitesimals was effectively criticized by Berkeley as so much hazy metaphysical mysticism. D'Alembert's visionary anticipation of the rigorisation of analysis was ahead of his time. Cauchy took first steps toward replacing infinitesimals by rigor and epsilontics, in particular giving a modern definition of continuity. Cauchy's false 1821 version of the "sum theorem" was corrected by him in 1853 by adding the hypothesis of uniform convergence. Weierstrass finally rigorized analysis and thereby eliminated infinitesimals from mathematics. Dedekind discovered "the essence of continuity", which is captured by his cuts. One of the spectacular successes of the rigorous analysis was the mathematical justification of Dirac's "delta functions". Robinson developed a new theory of infinitesimals in the 1960s, but his approach has little to do with historical infinitesimals. Lakatos pursued an ideological agenda of Kuhnian relativism and fallibilism, inapplicable to mathematics.

Each of the above ten claims is in error, as we argue in the next ten sections (cf. Crowe (1988)).

The historical fact of the dominance of the view of analysis as being based on the real numbers to the exclusion of infinitesimals, is beyond dispute. One could ask oneself why this historical fact is so; some authors have criticized mathematicians for adhering to an approach that others consider less appropriate. In the present text, we will *not* be concerned with either of these issues. Rather, we will be concerned with another issue, namely, why is it that traditional historical scholarship has been inadequate in indicating that alternative views have been around. We will also be concerned with documenting instances of tendentious interpretation and the attendant distortion in traditional evaluation of key figures from mathematical history.

Felix Klein clearly acknowledged the existence of a parallel, infinitesimal approach to foundations. Having outlined the developments in real analysis associated with Weierstrass and his followers, Klein pointed out in 1908 that

> The scientific mathematics of today is built upon the series of developments which we have been outlining. But an essentially different conception of infinitesimal calculus has been running parallel with this [conception] through the centuries (Klein 1908, p. 214).

Such a different conception, according to Klein, "harks back to old metaphysical speculations concerning the structure of the continuum according to which this was made up of [...] infinitely small parts" (ibid.). The pair of parallel conceptions of analysis are illustrated in Fig. 1.

A comprehensive re-evaluation of the history of infinitesimal calculus and analysis was initiated by Katz and Katz (2011a,b, 2012). Briefly, a philosophical disposition characterized by a preference for a sparse ontology has dominated the historiography of mathematics for

Fig. 1 Pair of parallel conceptions of the continuum (the thickness of the *top line* is merely conventional). The "thick-to-thin" *vertical arrow* "st" represents taking the standard part (see Appendix A for details)