Rigid Particle and its Spin Revisited

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Received August 23, 2006; revised October 2, 2006; Published online January 6, 2007

Communicated by Alwyn van der Merwe

The arguments by Pandres that the double valued spherical harmonics provide a basis for the irreducible spinor representation of the three dimensional rotation group are further developed and justified. The usual arguments against the inadmissibility of such functions, concerning hermiticity, orthogonality, behaviour under rotations, etc., are all shown to be related to the unsuitable choice of functions representing the states with opposite projections of angular momentum. By a correct choice of functions and definition of inner product those difficulties do not occur. And yet the orbital angular momentum in the ordinary configuration space can have integer eigenvalues only, for the reason which have roots in the nature of quantum mechanics in such space. The situation is different in the velocity space of the rigid particle, whose action contains a term with the extrinsic curvature.

KEY WORDS: rigid particle; quantum mechanics; quantization; spin angular momentum; double valued spherical harmonics; scalar product; renormalization.

1. INTRODUCTION

The theory of point particle whose action contains not only the length, but also the extrinsic curvature of the worldline has attracted much attention.\(^1\)\(^-\)\(^7\) Such particle, commonly called “rigid particle”, is a particular case of rigid membranes of any dimension (called “branes”). The rigid particle admits a wide class of solutions with one and two dimensional Zitterbewegung, and can, in particular, behave as a particle with spin. The spin occurs because, even if free, a rigid particle traces a worldline which deviates from a straight line. In particular, it can be a helical worldline.\(^5\) A treatment of various different cases related to possible rigid particle motions is given in refs. 3, 4. A more general discussion of the cases of

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the so-called “non-Galilean motion” or “Non Newtonian” can be found in ref. 8, and in a recent critical review of inert properties of classical relativistic point objects. There are objects, different from rigid particles, which can exhibit Zitterbewegung. Such is the Caldirola particle which may be regarded either as an extended object or a point particle with discrete time.

In the absence of an external field, the constants of motion of a rigid particle are the linear momentum $p_\mu$ and the total angular momentum $J_{\mu\nu}$ which is the sum of the orbital angular momentum $L_{\mu\nu}$ and the spin tensor $S_{\mu\nu}$. A complete exploration of the behaviour of rigid particles in the presence of external forces has not yet been done, so we do not know whether the square $S_{\mu}S_{\mu}$ of the Pauli-Lubanski vector $S_{\mu}$ is constant, as it is for a spinning particle. What has already been shown is that, in the presence of a gravitational field, the equation of motion for rigid particle has the same form as the well known Papapetrou equation.

In ref. 6 the free rigid particle whose action contained the square of the extrinsic curvature was considered. It was shown that the algebra of the (classical) Poisson brackets and the (quantum) commutators for such system resembled that of a spinning particle, and it was concluded that the rigid particle leaded to the Dirac equation. In refs. 13, 14 a counter argument occurred, namely, that the spin of the rigid particle was formally like the orbital angular momentum, with the only difference that it acted not in the ordinary configuration space, but in the space of velocities. Since orbital momentum is well known to posses integer values only, it was concluded that the rigid particle cannot have half-integer spin values. In the present paper we will challenge that conclusion.

A theoretical justification of why orbital angular momentum is allowed to have integer values only, and not half-integer, had turned out to be not so straightforward, and the arguments had changed during the course of investigation. Initially it was taken for granted that the wave function had to be single valued. Then it was realized that only experimental results needed to be unique, but the wave function itself did not need to be single valued. So Pauli found another argument, namely that the appropriate set of basis functions had to provide a representation of the rotation group. He argued that the spherical functions $Y_{lm}$ with half-integer $l$ failed to provide such representation.

Amongst many subsequent papers on the subject, those by Pandres are special: he demonstrated that the above assertion by Pauli was not correct. Pandres conclusion was that the functions $Y_{lm}$ with half-integer $l$ failed to provide such representation.

Following the common practice we will call $S_{\mu\nu}$ ‘spin tensor’, although this designation has to be taken with caution.