Classical and Non-relativistic Limits of a Lorentz-Invariant Bohmian Model for a System of Spinless Particles

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Abstract A completely Lorentz-invariant Bohmian model has been proposed recently for the case of a system of non-interacting spinless particles, obeying Klein-Gordon equations. It is based on a multi-temporal formalism and on the idea of treating the squared norm of the wave function as a space-time probability density. The particle’s configurations evolve in space-time in terms of a parameter $\sigma$ with dimensions of time. In this work this model is further analyzed and extended to the case of an interaction with an external electromagnetic field. The physical meaning of $\sigma$ is explored. Two special situations are studied in depth: (1) the classical limit, where the Einsteinian Mechanics of Special Relativity is recovered and the parameter $\sigma$ is shown to tend to the particle’s proper time; and (2) the non-relativistic limit, where it is obtained a model very similar to the usual non-relativistic Bohmian Mechanics but with the time of the frame of reference replaced by $\sigma$ as the dynamical temporal parameter.

Keywords Bohmian mechanics · Klein-Gordon equation · Relativistic quantum mechanics · Multi-temporal formalism · Space-time probability density · Conditional wave function

We would like to dedicate this work to our teacher Dr. Luis de la Peña Auerbach, who has always promoted the exploration of alternatives to the single orthodoxy in Quantum Mechanics.

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1 Introduction

Bohmian Mechanics is a non-local deterministic theory of particles in motion that seem to account for all the phenomena of non-relativistic quantum mechanics and solves the problem of the “collapse of the wave function” [1–9]. There are two physical entities in the theory: the wave function and the particles (which do have real trajectories in Bohmian mechanics). In the simple case of a system of spinless particles that do not interact electromagnetically with each other, Bohmian mechanics may be summarized as follows:

The wave function, \( \varphi(\vec{x}(1), \vec{x}(2), \ldots, \vec{x}(N), t) = R \exp[i \tilde{S}/\hbar] \) for \( N \) particles satisfies the usual Schröedinger equation:

\[
i \hbar \frac{\partial \varphi}{\partial t} = -\sum_{i=1}^{N} \frac{\hbar^2}{2m_i} \nabla^2_{(i)} \varphi + V \varphi, \tag{1.1}
\]

where \( V \) is the potential of interaction with an external electromagnetic field. In Bohm theory, however, (1.1) is not the complete description, but it is supplemented by the Bohm equation that describes how the particle velocities are determined by the wave function:

\[
\frac{d \vec{X}(i)}{dt} = \frac{1}{m_i} \nabla_{(i)} \tilde{S}(\vec{X}(1), \vec{X}(2), \ldots, \vec{X}(N), t), \tag{1.2}
\]

where the index \( i \) runs over all the particles \( (i = 1, \ldots, N) \). As it is common in the bohmian literature, in the previous equations, as in the whole of the paper, the generic variables \( \vec{x}(1), \vec{x}(2), \ldots, \vec{x}(N) \), on which the wave function depends, will be denoted by small letters, while the configurational variables \( \vec{X}(1), \vec{X}(2), \ldots, \vec{X}(N) \) (that is, the actual positions of the particles) will be denoted by capital letters.

J.S. Bell showed that any theory that makes the correct experimental predictions must be non local [2, 10], so the explicitly non-local character of Bohmian Mechanics should not be regarded as a defect of the theory. Nevertheless, it does bring the question of its relation to the Special Theory of Relativity. In other words, can a version of Bohm’s theory be Lorentz-invariant, in spite of its non-locality? Recently it has been suggested the possibility of reconciling Bohm’s theory and Relativity by the use of a multi-temporal formalism [11–18]. In the traditional formalism the wave function of a system of \( N \) particles depends on \( N \) spatial variables and a unique time, that is,

\[
\vec{x}(1), \vec{x}(2), \vec{x}(3), \ldots, \vec{x}(N), t.
\]

On the other hand, in the multi-temporal formalism the unique time is replaced by a set of \( N \) different times, one for each particle:

\[
\vec{x}(1), t(1), \vec{x}(2), t(2), \vec{x}(3), t(3), \ldots, \vec{x}(N), t(N) \tag{1.3}
\]

This is done with the aim of treating time on an equal footing with space, in the spirit of the Special Theory of Relativity [11–18]. Nikolic [12] has done a very interesting additional contribution by interpreting the squared norm of the wave function \( \psi \), in