MASSIVE GAUGE FIELDS AND THE PLANCK SCALE

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The present work is devoted to massive gauge fields in special relativity with two fundamental constants— the velocity of light, and the Planck length, so called doubly special relativity (DSR). The two invariant scales are accounted for by properly modified boost parameters. Within above framework we construct the vector potential as the $(1/2, 0)\otimes(0, 1/2)$ direct product, build the associated field strength tensor together with the Dirac spinors and use them to calculate various observables as functions of the Planck length.

Key words: special relativity, massive gauge fields, fundamental constants.

1. INTRODUCTION

In the standard theory of special relativity, Lorentz transformations preserve the energy-momentum dispersion relation of a particle observed from two different inertial frames according to

\[ E/c^2 - p_x^2 - p_y^2 - p_z^2 = (E'/c)^2 - p_x'^2 - p_y'^2 - p_z'^2 = m^2c^2. \] (1)

Lorentz transformations are covered by rotations in the three space-like planes $(p_x p_y)$, $(p_y p_z)$, and $(p_z p_x)$, on the one side, and by pseudo-rotations (i.e., rotations by an imaginary angle) in the $(E/c, p_z)$, $(E/c, p_y)$, and $(E/c, p_x)$ planes, on the other side. To be specific, for the text-book
example of $p_y' = p_y$, and $p_z' = p_z$, the boost parametrizes as

$$
\begin{pmatrix}
\frac{E'}{c} \\
p_z' \\
p_y'
\end{pmatrix} = 
\begin{pmatrix}
\cosh \phi \sinh \phi & 0 & 0 \\
\sinh \phi \cosh \phi & 0 & 0 \\
0 & 0 & 1
\end{pmatrix} 
\begin{pmatrix}
\frac{E}{c} \\
p_z \\
p_y
\end{pmatrix}.
$$

$$\cosh \phi = \frac{E}{mc^2}, \quad \sinh \phi = |\mathbf{p}|/mc.$$

Ordinary special relativity, to be referred to as SR in the following, shows up here through the dependence of the boost parameters on the velocity of light. Usually, velocities are given in units of $c$, and one sets $c = 1$.

In recent years, certain phenomena of Ultra High Energy Cosmic Rays (UHECR) seem to indicate that in the vicinity of the Planck length, ordinary special relativity may need an extension that accounts for the constancy of the Planck scale. To be more specific, in effect of collisions with the soft photons from the cosmic microwave background radiation, one expects cosmic ray protons, and cosmic gamma rays to slow down to energies below $E_p < 5 \times 10^{19}$ eV, and $E_\gamma < 20 \text{ TeV}$, respectively. The UHECR protons slow down basically because of pion-photo production, while the ultra high energy gamma rays lose energy due to electron-positron pair production. Yet, cosmic protons with energies $E_p > 5 \times 10^{19}$ eV (so called Greisen-Zatsepin-Kusmin (GZK) threshold value) as well as cosmic gamma rays with $E_\gamma > 20 \text{ TeV}$ still arrive at earth, and the GZK limit seems too low in comparison to data [1].

A possible solution to this so called cosmic ray problem has been advocated in Refs. [2,3]. According to Amelino-Camelia, quantum gravity effects may force deformations upon the energy-momentum dispersion relation as

$$E^2 \approx c^2 p^2 + c^4 m^2 + \lambda_p E p^2 + O(\lambda_p^2) .$$

Modifications of this type can allow for a higher value of the GZK limit [4-6]. Special relativities with two invariant scales are known as Doubly Special Relativity (DSR), a notion due to Ref. [2]. The major idea of such theories of space-time is to replace the linear parametrization of the boost by a non-linear function of the Planck length without changing the algebra of the Lorentz group [7]. Amelino-Camelia’s deformed energy-momentum dispersion relation results from the following non-linear boost parametrization (to be referred to as DSRa in the following)

$$\cosh \xi = \frac{e^{\lambda_p E} - \cosh(\lambda_p m)}{\sinh(\lambda_p m)} , \quad \sinh \xi = \frac{\lambda_p |\mathbf{p}| e^{\lambda_p E}}{\sinh(\lambda_p m)}.$$