ENERGY IN THE SCHWARZSCHILD-DE SITTER SPACETIME

Mustafa Salti and Oktay Aydogdu

Department of Physics, Faculty of Art and Science, Middle East Technical University, 06531, Ankara-Turkey
E-mail(s): musts6@yahoo.com, oktay231@yahoo.com

Received 3 December 2005; revised 13 December 2005

The energy (due to matter and fields including gravitation) of the Schwarzschild-de Sitter spacetime is investigated by using the Møller energy-momentum definition in both general relativity and teleparallel gravity. We found the same energy distribution for a given metric in both of these different gravitation theories. It is also independent of the teleparallel dimensionless coupling constant, which means that it is valid in any teleparallel model. Our results sustain that (a) the importance of the energy-momentum definitions in the evaluation of the energy distribution of a given spacetime and (b) the viewpoint of Lessner that the Møller energy-momentum complex is a powerful concept of energy and momentum.

Key words: energy, Schwarzschild-de Sitter spacetime, general relativity, teleparallel gravity, Møller’s prescription.

1. INTRODUCTION

Choosing the Schwarzschild gauge, the line-element can be written in the following form

\[ ds^2 = \Delta(r)dt^2 - \Delta^{-1}(r)dr^2 - r^2(d\theta^2 + \sin^2 \theta d\varphi^2), \]  

(1)

where \( \Delta(r) \) is an arbitrary (continuous, differentiable) function of \( r \). For spacetime with a single horizon (like Schwarzschild, de Sitter), \( \Delta(r) \) vanishes at one point, say, \( r = r_0 \). Near \( r_0 \), \( \Delta(r) \) can be \( \Delta(r) = \Lambda(r_0)(r-r_0) \), where \( \Lambda(r_0) \) is twice the surface gravity of the horizon. For spacetime with multiple horizons (like Schwarzschild-de Sitter), \( \Delta(r) \)
vanishes at more than one point, say, \( r = r_i \) (where \( i = 1, 2, 3, ..., n \)). Around each of these points one can expand \( \Delta(r) = \Lambda(r_i)(r-r_i) \), where \( \frac{\Lambda(r_i)}{2} \) is the surface gravity of each of these horizons.

The spherically coordinate of the Schwarzschild-de Sitter spacetime is defined by line-element given above, where

\[
\Delta(r) = 1 - 2M/r - r^2/l^2;
\]

here \( M \) is the mass of the black hole, and \( l^2 \) is related to the positive cosmological constant. The spacetime model has more than one horizon if \( 0 < \chi < 1/27 \) where \( \chi = M^2/l^2 \).

The black hole horizon \( r_h \) and the cosmological horizon \( r_c \) are located [1], respectively, at

\[
r_h = \frac{2M}{\sqrt{3\chi}} \cos \frac{\pi + \Xi}{3},
\]

\[
r_c = \frac{2M}{\sqrt{3\chi}} \cos \frac{\pi - \Xi}{3},
\]

where

\[
\Xi = \arccos(3\sqrt{3\chi}).
\]

It is of interest to investigate the energy distribution associated with this black hole model. We hope to calculate the same energy distribution in both general relativity and teleparallel gravity. This is the motivation of the present paper.

After the pioneering expression by Einstein [2] for the energy and momentum distributions of the gravitational field, many attempts have been proposed to resolve the gravitational energy problem [3, 4]. Except the definition of Møller, these definitions give meaningful results only if the calculations are performed in “Cartesian” coordinates. Møller constructed an expression which enables one to evaluate energy and momentum in any coordinate system. In general relativity, Virbhadra [5] using the energy and momentum complexes of Einstein, Landau-Lifshitz, Papapetrou and Weinberg for a general non-static spherically symmetric metric of the Kerr-Schild class, showed that all of these energy-momentum formulations give the same energy distribution as in the Penrose energy-momentum formulation. Vargas in [4] using the definitions of Einstein and Landau-Lifshitz in teleparallel gravity, found that the total energy is zero in Friedmann-Robertson-Walker spacetimes and his result is the same as calculated in general relativity [6]. Further, several examples of particular spacetimes have been investigated and different energy-momentum pseudo-tensors are known to give the same energy distribution for a given spacetime [7–9].

*Notations and conventions:* \( c = \hbar = 1 \), metric signature \((+,-,-,-)\), Greek indices and Latin ones run from 0 to 3. Throughout this paper, Latin indices \((i, j, ...\) represent the vector number, and Greek indices \((\mu, \nu, ...\) represent the vector components.