A WAY TO DISCOVER MAXWELL’S EQUATIONS THEORETICALLY

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The Coulomb force, established in the rest frame of a source-charge $Q$, when transformed to a new frame moving with a velocity $\mathbf{V}$ has a form $\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$, where $\mathbf{E} = \mathbf{E}'_{\parallel} + \gamma \mathbf{E}'_{\perp}$ and $\mathbf{B} = (1/c^2)\mathbf{v} \times \mathbf{E}$ and $\mathbf{E}'$ is the electric field in the rest frame of the source. The quantities $\mathbf{E}$ and $\mathbf{B}$ are then manifestly interdependent. We prove that they are determined by Maxwell’s equations, so they represent the electric and magnetic fields in the new frame and the force $\mathbf{F}$ is the well known from experiments Lorentz force. In this way Maxwell’s equations may be discovered theoretically for this particular situation of uniformly moving sources. The general solutions of the discovered Maxwell’s equations lead us to fields produced by accelerating sources.

Key words: Maxwell’s equations, Lorentz transformation, Coulomb’s law.

1. INTRODUCTION

Maxwell’s equations:

\[
\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0},
\]

\[
\nabla \times \mathbf{B} - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \frac{\mathbf{j}}{c^2 \varepsilon_0},
\]

\[
\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0,
\]

\[
\nabla \cdot \mathbf{B} = 0,
\]

(1) (2) (3) (4)
and the Lorentz force law

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$ \hspace{1cm} (5)

are generalizations based on many different experiments on the forces between electric charges and currents. The Lorentz force law is usually introduced as an assumption separate from the field equations. Nevertheless the entire structure appears to be Lorentz covariant. Since the theory of relativity establishes some relationships among Maxwell’s equations and between electric and magnetic fields, this suggest that the equations of classical electrodynamics could be derived from fewer experimental data by using relativistic arguments.

In this paper we want to reveal such a possibility. It is our desire to introduce Maxwell’s equations in a manner which more clearly exhibits the role of special relativity. Namely, we show that to find out Maxwell’s equations it is enough to know from experiments only Coulomb’s law (established in the rest frame of the source-charge). Using the Lorentz transformation of coordinates we are able to determine the force and its components (the fields $\mathbf{E}$ and $\mathbf{B}$) produced by the source-charge when it is in motion with constant velocity. We prove that defined within this formalism fields $\mathbf{E}$ and $\mathbf{B}$ undergo Maxwell’s equations. In this way the experimental way of discovering Maxwell’s equations may be replaced to some extend by a theoretical study.

Certainly, our method cannot be applied to accelerating sources because we do not know the Lorentz transformation between accelerating frames. Surprisingly Maxwell’s equations we derive for the special case, i.e., charges in uniform motion, have also much more general solutions depending on the acceleration of sources. The discovered theoretically laws lead us then to the completely general electromagnetic fields. Nevertheless one must be aware that while the physical correctness of the solutions of Maxwell’s equations for uniformly moving charges is logically guarantied by our reasoning (provided our space-time structure is Minkowskian), validity of the general solutions requires experimental verification.

Let us explain it in more detail in the context of the known views on the issue. It may seem that the results presented in this paper are in contradiction to the opinion of Feynman. In his lectures he wrote that the statement that ”all of electrodynamics can be deduced solely from Lorentz transformation and Coulomb’s law (...) is completely false.” [1] Next he explains that to obtain Maxwell’s fields one must assume the retardation of interactions and that the electromagnetic potentials do not depend on the acceleration of sources. The textbook by W. G. V. Rosser [2] is the most famous work in which the development of Maxwell’s fields is performed on the basis of these assumptions. The equivalent set of assumptions is used by Frisch and Wilets [3] in their paper. The standpoint we represent is essentially different. First of all, we do not derive directly Maxwell’s fields but Maxwell’s equations.