WHAT’S WRONG WITH THIS REBUTTAL?

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A recent rebuttal to criticism of Bell’s analysis is shown to be defective by fault of failure to consider all hypothetical conditions input into the derivation of Bell Inequalities.

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1. THE DISPUTE

On the pages of Found. Phys. [1], under the title: “What’s wrong with this Criticism?,” N. D. Mermin rebutted criticism of Bell’s Theorem and an analysis by K. Hess and W. Philipp [2]. The latter authors hold that Bell’s analysis of EPR-inspired experiments testing the contention that quantum mechanics could be “completed,” i.e., rendered a deterministic theory instead of only a probabilistic one, is fatally flawed. They argue that Bell failed to consider time variable correlations and subsequently failed to find structure permitting a local realistic interpretation of the results of EPR experiments.

While this is the context of the larger dispute, the actual point of contention for Mermin was a much narrower, although equally potent, sub-argument that Hess and Philipp mentioned along the way. It is essentially this: For technical reasons, data taken in feasible experiments cannot meet all requirements in the input into derivations of Bell Inequalities. This is an old observation, although its many renditions are not always easily recognized as being the same issue.

To crystallize the crucial points, recall that Bell’s analysis ostensibly proves that for all local realistic theories a certain expression,
in Mermin’s notation denoted by $\Gamma$, satisfies

$$|\Gamma| \leq 2. \quad (1)$$

Now $\Gamma$, as is easily seen from its derivation (which is very well known and will not be reiterated here; see [3]), for EPR-type experiments is comprised of a particular sum of terms, where each one is the sum of the products of the outcomes in each arm for a given combination of the polarizer settings (or Stern-Gerlach field directions if the experimental objects are electrons). In so far as two different settings are considered for each arm, there are then four combinations so that $\Gamma$ can be written as

$$\Gamma = \frac{1}{N} \sum_j [a_a(j)b_b(j) + a_a(j)b_c(j) + a_d(j)b_b(j) - a_d(j)b_c(j)]. \quad (2)$$

It is just here that Hess and Philipp, as have others before them,\(^1\) raise an objection. It is that for the different settings of the polarizers, that is, for the various of the four combinations or each term alone, one has essentially four different experiments and that it is not legitimate to mix the data and analyze it as if it came from a single experiment.

2. MERMIN’S CONTENTION

In response, Mermin’s rebuttal consists of asserting that Eq. (2) can also be written

$$\Gamma = \frac{1}{N_{ab}} \sum_{j \in X_{ab}} a_a(j)b_b(j) + \frac{1}{N_{ac}} \sum_{j \in X_{ac}} a_a(j)b_c(j)$$

$$+ \frac{1}{N_{db}} \sum_{j \in X_{db}} a_d(j)b_b(j) - \frac{1}{N_{dc}} \sum_{j \in X_{dc}} a_d(j)b_c(j)], \quad (3)$$

and that, in a sufficiently long run, that is large enough $N$, “each of the four choices for $xy$ the $N_{xy}$ indices $j$ appearing in $X_{xy}$ constitute a random sample of the full set $j = 1 \ldots N$, each $j$ having the probability $1/4$ of appearing in $X_{xy}$. So by standard sampling theory . . . .”

3. A LACUNA IN THE REBUTTAL

The point of this rebuttal is necessary but not sufficient, however. The data collected in four equal length sub-runs, one each for each polarizer

\(^1\)Recent studies by Adenier [4] and Sica [5] explicitly make the same point. Apparently, de la Peña, Cetto and Brody [6] were first to recognize the significance of the relevant structure.