CONTACT INTERACTION OF CRACK FACES UNDER OBLIQUE INCIDENCE OF A HARMONIC WAVE

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Abstract: The contact interaction of the opposite faces of a crack under oblique incidence of a harmonic wave is considered. The problem is solved by the method of boundary integral equations using an iterative algorithm. The contact forces and the displacement discontinuity on the crack faces are studied for different values of the friction coefficient.

Keywords: Harmonic loading, interaction of crack faces, contact force, displacement discontinuity, friction.

Introduction. Under dynamic loading the opposite crack faces interact with each other, altering significantly the stress distribution near the crack tips. The contact behaviour is very sensitive to the material properties of contacting surfaces and to the external loading. The shape and extent of the contact region will change in time and must be determined as a part of solution. Such dependences make the contact crack problem highly non-linear. As the consequence, in the majority of the contact crack problems, the real stress-strain distribution is neglected due to the difficulties of finding the appropriate solution. The reviews of the commonly used methods and the results obtained in the fracture mechanics for cracked solids are given by Cherepanov (1979), Balas et al (1989), Zhang and Gross (1998), Guz and Zozulya (2001).

1. Problem statement. Consider a penny-shaped crack located within the homogeneous, isotropic, linear elastic medium. In this study we examine the crack which initially is fully closed, therefore, the initial opening is \( h_0(x) = 0 \). The crack is described by the corresponding middle surface \( \Omega = \{ x_1^2 + x_2^2 \leq a^2; \ x_3 = 0 \} \), where \( a \) is the crack radius.
A time-harmonic tension-compression wave propagates in the medium. It is defined by the following scalar function

\[ \Phi(x, t) = \Phi_0 \exp(i(k_1(x, \cos \alpha + x_2 \sin \alpha) - \omega t)), \]

where \( \omega = 2\pi/T \) is the wave frequency, \( T \) is the oscillation period, \( \Phi_0 \) is the amplitude, \( k_1 = \omega/c_1 \) is the generalised wave number. The velocity of the longitudinal wave is \( c_1 = \sqrt{(\lambda + 2\mu)/\rho} \); \( \mu \) and \( \lambda \) are the Lame elastic constants; \( \rho \) is the material density; \( \alpha \) is the angle of incidence of the wave.

The load on the crack faces due to the incident wave has the following form

\[ p^i(x, t) = \begin{cases} 
-\Phi_0 k_1^2 \mu \sin 2\alpha (\cos(k_1 x_1 \cos \alpha \cos(\omega t) - \sin(k_1 x_1 \cos \alpha)\sin(\omega t)) \\
0 \\
-\Phi_0 k_1^2 (\lambda + 2\mu \sin^2 \alpha) (\cos(k_1 x_1 \cos \alpha \cos(\omega t) - \sin(k_1 x_1 \cos \alpha)\sin(\omega t)) 
\end{cases}, \]

\[ x \in \Omega, \ t \in [0; T]. \]

Under the external dynamic loading the opposite crack faces move with respect to each other and the corresponding displacement is given by the discontinuity vector \( \Delta u(x, t) \). The contact interaction results in the appearance of the contact force \( q(x, t) \) in the contact region.

We impose the Signorini constraints (1) for the normal components of the contact force and the displacement discontinuity vectors

\[ \Delta u_1(x, t) \geq -h_0(x), \ q_1(x, t) \geq 0, \ (\Delta u_1(x, t) + h_0(x))q_1(x, t) = 0, \]  \hspace{1cm} (1)

i.e. there is no interpenetration of the opposite crack faces and the contact force is unilateral. Additionally we assume that the contact interaction satisfies the Coulomb friction law (2):

\[ \left| q_1(x, t) \right| < k, q_3(x, t) \Rightarrow \frac{\partial \Delta u_1(x, t)}{\partial t} = 0; \]

\[ \left| q_1(x, t) \right| = k, q_3(x, t) \Rightarrow \frac{\partial \Delta u_1(x, t)}{\partial t} = -\frac{q_1(x, t)}{\left| q_1(x, t) \right|} \left| \frac{\partial \Delta u_1(x, t)}{\partial t} \right|, \]  \hspace{1cm} (2)

where \( x \in \Omega, t \in [0; T] \).

2. Methodology. If the contact interaction of crack faces is taken into account, the resulting process is a steady-state periodic process, but not a harmonic one. As a result, components of the stress-strain state due to the reflected waves cannot be represented as a function of coordinates multiplied by an exponential function. According