MICROFIBRIL-BASED ESTIMATES OF THE BALLISTIC LIMIT OF MULTI-LAYERED FABRIC SHIELDING

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Abstract. An expression relating the dominant mechanisms for the energy loss of a projectile, as it penetrates a multilayered ballistic fabric shield, is developed. The relation is a function of the projectile mass, initial velocity and fabric microscale properties, such as microfibril stiffness and critical stretch. Using the derived expression, the number of fabric layers needed to stop a projectile is determined. The analytical result compares quite well to experiments.

Keywords: multilayered ballistic fabric, microscale properties


2 Stretching and rupture. We initially consider the rupture of a single “yarn” in the fabric (Figure 1). We denote \( \rho_0 \) as the initial (undeformed) density of the yarn and \( A_0 \) is the initial cross-sectional area. During the stretching to final rupture, the yarn is assumed to attain a linear (symmetric) velocity profile given by \( (0 \leq x \leq \frac{L_0}{2}) \)

\[
v(x) = v_p \frac{x}{L_0},
\]

(1)

where \( v_p \) is the velocity of the projectile (Figure 1). We assume the that contact area of the projectile/fabric is sufficiently small with respect to the target size so that it can be considered as a point-load during this stage of the analysis. Later, the dimensions of the projectile will be taken into account. By integrating the differential kinetic energy, we obtain
\[
2 \int_0^{L_o} \frac{1}{2} \rho_o A_o (v(x))^2 \, dx = \frac{\rho_o A_o L_o}{6} (v_p)^2.
\]

(2)

An energy balance yields

\[
\frac{1}{2} m_p (v_p^{(i)})^2 - \underbrace{w(U^*)}_{\text{kinetic energy after the } i\text{th sheet}} = \underbrace{\frac{1}{2} m_p (v_p^{(i+1)})^2 + \frac{\rho_o A_o L_o}{6} (v_p^{(i+1)})^2}_{\text{kinetic energy after the } i+1\text{th sheet}} + \underbrace{\text{stored elastic energy}}_{\text{fabric kinetic energy}},
\]

(3)

where the stretch throughout the yarn (on either side of the projectile) has been approximated as being uniform, due to the assumed mode of deformation (Figure 1).

The stored energy at failure is denoted \( w(U^*) \), where \( U^* \) is the critical stretch ratio at failure. The stretch ratio is defined as \( U_{\text{def}} = \frac{L}{L_o} \), where \( L \) is the stretched length and \( L_o \) is the original unstretched length. Equation 4 may be written in the form of a recursion \( (i=\text{sheet number}) \)

\[
v_p^{(i+1)} = \sqrt{\alpha (v_p^{(i)})^2 - \beta},
\]

(4)

where

\[
\alpha = \frac{m_p}{m_p + \frac{\rho_o A_o L_o}{3}} \quad \text{and} \quad \beta = \frac{2w(U^*)}{m_p + \frac{\rho_o A_o L_o}{3}}.
\]

(5)

For moderate finite strains (the case here), the response of the thin one-dimensional yarn can be accurately described by a constitutive law of the form \( S = IE \), where \( S \) is the second Piola-Kirchhoff stress, \( IE \) is Young’s modulus and \( E \defeq \frac{1}{2} (U^2 - 1) \) is the Green-Lagrange strain. The quantity of interest here is the stored energy \( w(U^*) = \frac{L_o A_o}{2} \frac{IE}{2} \left( \frac{(U^*)^2 - 1}{2} \right)^2 \) at final rupture, where \( W(U^*) \defeq \frac{1}{2} IE \left( \frac{(U^*)^2 - 1}{2} \right)^2 \) is the stored energy per unit volume. We shall use this simple stored energy function in the analysis that follows. However, we note that other material models could easily be employed without any complication.

\[ \text{CONTACT} \quad \text{STRETCHING} \quad \text{PENETRATION} \]

\[ \text{Figure 1: LEFT: Sequence of events. RIGHT: The velocity profile.} \]