Analytical prediction of the phase transformation onset zone at a crack tip of a shape memory alloy exhibiting asymmetry between tension and compression

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Abstract Shape Memory Alloys (SMAs) such as NiTi exhibit stress induced martensitic phase transformation. The purpose of this paper is to provide a better understanding of SMA (such as NiTi) fracture behavior, by considering the vicinity of the crack tip where the transformation occurs. This analysis integrates the asymmetry between tension and compression in an analytical prediction of the surface of phase transformation around the crack tip for loading modes 1, 2, 3 and mixed 1+2. The influence of the asymmetry between tension-compression is more important in plane stress conditions than in plane strain conditions, particularly for mode 1 loading. In order to validate this model, we are currently setting up an experimental investigation to observe strain localization during crack propagation (transformation and martensitic saturation regions) on NiTi thin sheets.

Keywords Fracture · Crack · Phase transformation · Shape memory alloys · Transformation surface

1 Introduction

Shape Memory Alloys (SMAs) are potential materials for use in smart structures, actuators, medical devices and aeronautical materials. This is because of the very large recoverable strains (on the order of 8% for equiatomic NiTi) associated with their superelastic or pseudoelastic behaviour. The extended use of SMA elements, which are sometimes subjected to rather complex loadings, raises the issue of the service life of systems and leads us to investigate SMA fracture and/or fatigue damage. As discussed by Daly et al. (2007), fatigue and fracture behaviours, and their possible consequences on patients’ health, are of a great concern in the medical industry, where NiTi is widely used for medical devices like stents.

Pseudoelasticity is associated with a martensitic phase transformation (MPT), induced by an applied load, between high-symmetry austenite and low-symmetry martensite. SMAs accommodate the applied stress through MPT in which martensite variants (twinned or untwinned martensite) are created. The untwinned martensite differs in its crystallographic orientations. The mechanism of the MPT is well understood for single crystal (Miyazaki et al. 1983; Patoor et al. 2006) but not completely for polycrystal.

The linear elastic fracture mechanics (LEFM) theory provides the stress field around the crack tip, depending on the loading mode (1, 2, 3 or mixed). Theoretically, the stress field around the crack tip is unbounded. However, whereas other elastic-plastic materials undergo yielding at the crack tip, SMAs initiate an MPT in the vicinity of the crack tip where a phase transformation zone (from austenite through martensite states) appears locally.
In this case, the crack tip region is governed by the stress field associated with the SMAs pseudoelastic behavior. For instance, Yi and Gao (2000) investigated numerically the SMAs fracture toughening mechanism under mode I loading and showed that the MPT increases the toughness and decreases the crack tip intensity factor. Wang et al. (2005) examined the stress induced martensite near the crack tip of a CT specimen by an FEM calculation. The formation of stress induced martensite in front of a crack tip has similarities with the formation of a plastic zone in front of a crack tip in a material which undergoes plastic deformation. The amount of martensite and the extension of the transformed region in front of a sharp crack increases with the growing crack length and are load path dependent.

Actually, what is lacking, mainly, are experimental observations of the phase transformation zones around the crack tip. We can cite experimental work about the fracture toughening behaviour of a tube (Robertson and Ritchie 2007). Robertson and Ritchie measured in-situ three dimensional strains, phases and crystallographic alignment ahead of a growing fatigue crack (100 cycles; Robertson et al. 2007). Their measurements reveal that the majority of austenite grains were subjected to only 0.5–1.0% elastic strain despite a macroscopic superelastic strain recovery of 6–8% associated with the MPT.

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Daly performed a tensile test on an edge-crack specimen of austenitic NiTi using an in-situ optical technique to examine the shape and the size of phase transformation zones (Daly et al. 2007). They measured the fracture toughness $K_{ic}$ at room temperature for fine grained polycrystalline nitinol sheets.

And lastly, Freed and Banks-Sills predicted numerically the shape of the initial transformation zone around the crack tip under a mode 1 loading, for plane stress or plane strain conditions (Freed and Banks-Sills 2007). With this aim in mind, they used a phenomenological model proposed by Panoskaltsis et al. (2004). This model considers the phase transformation surface as a Huber-von Mises type surface without taking into account the asymmetry between tension and compression in a tridimensional model formulation that Vacher and Lexcellent highlighted in the past (Vacher and Lexcellent 1991).

This present work aims at including this important “stress directional effect” for the transformation surface predictions. Obviously, the yield surfaces determination is associated with proportional loading in each case. The three classical fracture modes, then a mixed mode, are investigated for plane stress and plane strain conditions.

## 2 General formulation of the phase transformation yield surfaces

The aim of this part is the development of phase transformation yield surfaces.

Let us consider the second order symmetric tensor $a$ and the deviatoric tensor $S_a$ defined by

$$ S_a = a - \frac{1}{3} tr(a) I $$

(1)

where $I$ is the identity tensor. Two dots : denotes the scalar product and $tr(.)$ denotes the trace operator, so that $a : b = tr(a^T b)$ so that for every second order tensors $a$ and $b$ and $|a| = \sqrt{tr(a^T a)}$.

Classically, the Huber-Von Mises equivalent stress $\bar{\sigma}$ is defined as $\bar{\sigma} = \kappa |S_a|$, with $\kappa \equiv \frac{\sqrt{2}}{2}$ the normalisation parameter. The Lode invariant $y_\sigma$ is defined by Raniecki and Lexcellent (1998)

$$ y_\sigma = \frac{27 det(S_q)}{2 \bar{\sigma}^3} = \frac{6\kappa}{|S_\sigma|^3} det(S_q) $$

(2)

In this analytical formulation, the three classical fracture loading modes are predicted: opening mode 1, shear plane mode 2, and shear antiplane or tearing mode 3.

Whatever the chosen mode $I = 1, 2$ or $3$, the stress tensor $\sigma_{M(r,\theta)}$, obtained for Linear Elastic Fracture Mechanics (LEFM) theory, can be decomposed as

$$ \sigma_{M(r,\theta)} = \chi_I(r,\theta) q I(\theta) $$

(3)

where $(r, \theta)$ are the polar coordinates centered at the crack tip point, as shown in Fig. 1, and

$$ \chi_I(r) = \frac{K_I}{\sqrt{2\pi r}} > 0 $$

(4)

depending on the loading mode $K_I = K_1, K_2$ or $K_3$. Figure 1 diagram of polar coordinates reference frame

Thus,

$$ S_\sigma = \chi_I(r) S_q(\theta) $$

(5)

and the Lode invariant $y_\sigma$ is independent of $r$

$$ y_\sigma(\theta) = \frac{6\kappa}{|S_q(\theta)|} det(S_q(\theta)) $$

(6)

The Huber-Von Mises equivalent stress $\bar{\sigma}$ can be written as

$$ \bar{\sigma} = \kappa \chi(r) |S_q(\theta)| = \kappa \frac{K_I}{\sqrt{2\pi r}} |S_q(\theta)| $$

(7)