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The stability of Killing–Cauchy horizons in colliding plane wave space-times

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Abstract It is confirmed rigorously that the Killing–Cauchy horizons, which sometimes occur in space-times representing the collision and subsequent interaction of plane gravitational waves in a Minkowski background, are unstable with respect to bounded perturbations of the initial waves, at least for the case in which the initial waves have constant aligned polarizations.

Keywords Gravitational wave · Horizon · Cauchy problem

1 Introduction

Many classes of explicit exact solutions are known which model the collision and subsequent interaction between shock-fronted plane gravitational waves which propagate and collide in a Minkowski background (for reviews see [1] or [2]). In all these solutions, some kind of singularity always appears in the interaction region. This is generally a spacelike curvature singularity (like the time-reverse of an initial cosmological singularity). However large classes of solutions with an infinite number of parameters exist in which the scalar polynomial curvature singularity is replaced by a horizon. In these cases, the space-time can be extended through the horizon, but the extension is not unique. It has become widely believed that such Killing–Cauchy horizons are unstable with respect to small changes in the initial data. For example, Yurtsever [3] has shown that they are unstable with respect to the addition of some perturbative linear field which preserves the $G_2$ symmetry. However, this result does not answer the question of whether they are unstable with respect to variations in the approaching purely gravitational waves.
when the vacuum field equations are satisfied exactly. It is the purpose of the present paper to investigate this question in detail.

It must first be pointed out that, in the vast amount of work that was undertaken on this topic in the 1970s and 1980s, and in all the exact solutions that were then produced, the approach was adopted of first solving the field equations in the interaction region. Once a family of such solutions had been obtained, the free parameters were constrained to satisfy the junction conditions that have to be imposed in order to extend the solution to the prior regions. Thus, the approaching waves which physically give rise to the solutions were only determined once the solution had been obtained. Within this context, it was argued that those solutions which contained horizons were unstable with respect to perturbations of the initial data. However, it was not then appreciated that the perturbations which transform a Killing–Cauchy horizon to a scalar polynomial curvature singularity also normally introduce singularities in the initial waves prior to their collision. The question therefore still needs to be addressed as to whether or not the horizons are stable with respect to regular (bounded) perturbations of physically acceptable initial waves.

It is only very recently that techniques have been developed by which colliding plane wave solutions can be explicitly constructed from their characteristic initial data representing the approaching waves [4–8]. It is therefore only now that tools are available to reconsider this question. In this paper, only the linear vacuum case in which two approaching gravitational waves have constant aligned polarizations will be considered. This situation is much easier to analyse, but it already demonstrates the essential features of the physical situation. The basic result, not surprisingly, is that the Killing–Cauchy horizons that sometimes appear in colliding plane wave space-times are unstable with respect to bounded perturbations of the initial waves that generate these solutions.

2 Initial data

Colliding plane wave space-times are naturally divided into four regions as indicated in Fig. 1. It is found to be convenient to use two future-pointing null coordinates $u$ and $v$ throughout the space-time. The four distinct regions can then be identified as those that are separated by two null hypersurfaces taken as $u = 0$ and $v = 0$ which represent the wavefronts of the approaching waves.

The background region I ($u < 0$, $v < 0$) is a flat vacuum represented by the line element

$$ds^2 = 2\, du\, dv - dx^2 - dy^2.$$  \hspace{1cm} (1)

Region II ($u \geq 0$, $v < 0$) contains one of the approaching plane waves. If this wave has constant (linear) polarization, it can be described by a metric in the Brinkmann form

$$ds^2 = 2\, du\, dr - dX^2 - dY^2 + h_+(u)(X^2 - Y^2)\, du^2,$$  \hspace{1cm} (2)

where $h_+(u) = \Psi_{44}(u)$, which is the only non-zero component of the Weyl tensor (relative to an appropriate tetrad). This arbitrary function explicitly represents the profile of the plane gravitational wave as a function of the retarded time $u$. 