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Wave mechanics and general relativity: a rapprochement

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Abstract Using exact solutions, we show that it is in principle possible to regard waves and particles as representations of the same underlying geometry, thereby resolving the problem of wave-particle duality.

Keywords Wave-particle duality

1 Introduction

Wave-particle duality is commonly presented as a conceptual conflict between quantum and classical mechanics. The archetypal example is the double-slit experiment, where electrons as discrete particles pass through a pair of apertures and show wave-like interference patterns. However, particles and waves can both be given geometrical descriptions, which raises the possibility that these behaviours are merely different representations of the same underlying geometry. We will give a brief discussion involving exact solutions of extended geometry, to show that particles and waves may be the same thing viewed in different ways.

Certain technical results will be needed below. (Those readers more interested in results than method may like to proceed to Sect. 2.) The basic idea is that waves and particles are different coordinate representations of the same geometry, or isometries [1–5]. Even in special relativity, which frequently uses as a basis four-dimensional Minkowski space ($M_4$), we can if we so wish change the form of the metric by a change of coordinates (or gauge). Thus, $M_4$ is actually isometric to the Milne universe, which is often presented as a Friedmann-Robertson-Walker (FRW) model with negative 3D or spatial curvature in general relativity [4]. While the metrics may look different, their equivalence is shown by the fact that in both
cases the density and pressure of matter are zero as determined by the field equations. The latter in 4D read $R_{\alpha\beta} - R g_{\alpha\beta}/2 + \Lambda g_{\alpha\beta} = 8\pi T_{\alpha\beta}$ ($\alpha, \beta = 0, 123$ for time and space, where the speed of light and the constant of gravity have been set to unity). Here $R_{\alpha\beta}$ is the Ricci tensor, $R$ is the Ricci scalar, $g_{\alpha\beta}$ is the metric tensor, $\Lambda$ is the cosmological constant and $T_{\alpha\beta}$ is the energy-momentum tensor.

While certain wave-like solutions of the latter equations are known [3], none has the properties of the deBroglie waves which are commonly used to describe the energy ($E$) and spatial momenta $p_{123}$ of particles in wave mechanics. Symbolically, these have wavelengths $\lambda^0 = h/E$, $\lambda^1 = h/p_1$ etc., where $h$ is Planck’s constant (which may also be set to unity). However, solutions of the field equations are known with deBroglie-like waves in dimensionally-extended gravity [5–7]. The latter is fundamentally Einstein’s theory of general relativity, extended to $N (> 4)$ D, in order to unify gravity with the interactions of particle physics. The basic extension is to $N = 5$, where Campbell’s theorem ensures that any solution of the 5D field equations in vacuum is also a solution of the 4D field equations with matter [1, 8]. That is, we can always recover a solution of the 4D equations noted above from the 5D equations, which in terms of the extended Ricci tensor are just $R_{AB} = 0$ ($A, B = 0, 123, 4$). There are many exact solutions known of these equations, whereby the extended version is known to agree with observations, both in regard to the solar system [5, 9] and cosmology [5, 10]. Several are relevant to the present project [11–14]. For example, the Billyard solution [14] has a metric coefficient for the 3D or spatial part which naturally represents the 3D or momentum component of a deBroglie wave [15]. It is a remarkable solution, in that it is not only Ricci-flat ($R_{AB} = 0$) but also Riemann-flat ($R_{ABCD} = 0$). That is, it represents a flat 5D space, which by virtue of Campbell’s theorem satisfies Einstein’s 4D equations, and has a 3D deBroglie wave. However, it is deficient in some respects as regards the present project, notably in that it has a signature $(+−−−−)$ which is at variance with the one $(+−−−−)$ indicated by particle physics. The latter subject is constrained by Lorentz invariance and experiments related to this. There is a relation between the energy $E$, 3-momentum $p$ and rest mass $m$ of a particle, which is regarded as standard because it is closely obeyed in experiments (see Ref. [16] for a review). Namely,

$$E^2 = p^2 + m^2. \quad (1)$$

This is a strong constraint on any attempt to construct a geometric relation between the particle and wave descriptions of matter. From the viewpoint of a theory like general relativity, (1) is perhaps not surprising, in that it can be understood as a consequence of multiplying a constant $m$ onto the conventional condition for normalizing the 4-velocities, viz $u^a u_a = 1$. (Here, $u^a \equiv dx^\alpha/ds$ where the 4D coordinates $x^\alpha$ are related to the proper time $s$ and the metric tensor $g_{\alpha\beta}$ via $ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$.) From the viewpoint of a dimensionally-extended theory, (1) is also not so surprising, in that it follows for a wide range of metrics. The latter involve two aspects. First, the coordinates should be “canonical” in 5D, which means that the interval can be written as $dS^2 = (l/L)^2 ds^2 - dl^2$ where $x^4 \equiv l$, so that the extra coordinate plays the role of particle mass and the Weak Equivalence Principle is obeyed [17, 18]. Second, the paths of particles (or waves) should be null, so that a photon-like object in 5D appears as a massive object in 4D [19–21]. This latter condition enables us to cast our project into a new form: we are asking