LETTER

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Ultimate gravitational mass defect

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Abstract We present a new type of gravitational mass defect in which an infinite amount of matter may be bounded in a zero ADM mass. This interpolates between effects typical of closed worlds and $T$-spheres. We consider the Tolman model of dust distribution and show that this phenomenon reveals itself for a solution that has no origin on one side but is closed on the other side. The second class of examples corresponds to smooth gluing $T$-spheres to the portion of the Friedmann-Robertson-Walker solution. The procedure is generalized to combinations of smoothly connected $T$-spheres, FRW and Schwarzschild metrics. In particular, in this approach a finite $T$-sphere is obtained that looks for observers in two $R$-regions as the Schwarzschild metric with two different masses one of which may vanish.

Keywords ADM-mass · Cosmology

1 Introduction

One of the remarkable features of general relativity is a gravitational mass defect because of which a total ADM mass $m$ measured of an external observer at infinity may significantly differ from the proper mass $M$ – total amount of matter contained in the system, $m < M$ [1]. There exist two distinct situations of extremely strong gravitation binding. (i) It can arise in a semi-closed world connected to the external region via “throat”. When this throat shrinks, a completely closed world appears separated from the external region, its mass $m$ tending to zero (see monographs [2–5] and references therein). (ii) Another type of the gravitation mass defect is inherent to systems without the centre of symmetry that are...
able to accumulate an infinite amount of matter but reveal themselves for an external observer as a body of a finite mass \( m \) [6]. In particular, this is the property typical of so-called \( T \)-spheres [7–9] (see below) which in the case of the dust source generalize the Kantowski–Sachs class [10] of exact solutions. The mass \( m \) for such models is constant but the \( T \)-region can extend infinitely long. (We adhere the classification and terminology introduced in [11]: if the gradient of the areal radius \( R \) is space-like, it is called \( R \)-region, if it is time-like, it is called \( T \)-region. In doing so it is implied that this gradient does not identically vanish).

Being glued to a vacuum region, such a system reveals itself in the outer space as a "\( T \)-sphere" of a finite mass. Thus, in the case (i) we are faced with a finite \( M \) and zero \( m \), in the case (ii) \( M \) is infinite, \( m \) is finite. It was specially stressed in [9] that the nature of the phenomenon under discussion in these two situations is qualitatively different. The basic aim of the present note is to point out that, nevertheless, both cases can be combined in such a way that they lead to a new type of the gravitational mass defect which is the most possible strong one: an infinite \( M \) corresponds to a zero \( m \).

2 Preliminaries: Tolman model and definition of mass

For simplicity, we restrict ourselves by perfect dust that admits exact solutions [12]. We begin with short description of their properties. As for dust there exists the frame which is simultaneously synchronous and comoving, the metric under the assumption of spherical symmetry can be cast into the form

\[
\begin{align*}
    ds^2 &= -d\tau^2 + b^2(\chi, \tau)d\chi^2 + R^2(\chi, \tau)d\omega^2, \\
    d\omega^2 &= d\theta^2 + d\phi^2 \sin^2 \theta.
\end{align*}
\] (1)

Usually, it is implied that the derivative \( R' \) does not vanish identically (hereafter, prime and dot denote derivatives with respect to \( \chi \) and \( \tau \), correspondingly). Then the model admits known Lemaître-Tolman-Bondi (LBT) family of solutions which describe an inhomogeneous collapse of dust (or its time reversal). However, there is also one more brunch that arises as a special solution of Einstein equations with the areal radius \( R = R(\tau) \) not depending on a spatial coordinate [7, 8, 13]. The special solution under discussion (called "\( T \)-spheres" or "\( T \)-models" in [7, 8]) possesses a number of unusual properties. Thus, there are two qualitatively different branches of the metric of the type (1).

2.1 LBT solutions

The LBT solutions can be represented in the form

\[
b^2 = \frac{R'^2}{1 + f(\chi)}, \tag{2}
\]

where \( f(\chi) \) should satisfy the inequality \( f + 1 \geq 0 \), otherwise being arbitrary. The time evolution is governed by equation

\[
R^2 = \frac{F(\chi)}{R} + f, \tag{3}
\]