The parametric transition of strange matter rings to a black hole

Hendrick Labranche · David Petroff · Marcus Ansorg

Received: 8 May 2006 / Accepted: 22 November 2006 / Published online: 16 December 2006
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Abstract It is shown numerically that strange matter rings permit a continuous transition to the extreme Kerr black hole. The multipoles as defined by Geroch and Hansen are studied and suggest a universal behaviour for bodies approaching the extreme Kerr solution parametrically. The appearance of a ‘throat region’, a distinctive feature of the extreme Kerr spacetime, is observed. With regard to stability, we verify for a large class of rings, that a particle sitting on the surface of the ring never has enough energy to escape to infinity along a geodesic.

Keywords Black hole · Stationary and axisymmetric spacetimes · Rings · Numerical solutions · Strange matter

1 Introduction

In this paper, we consider in detail the parametric transition of a strange matter ring to a black hole. In [1,2], necessary and sufficient conditions for a quasistationary transition were presented and it was proved that an extreme Kerr
black hole necessarily results. Using the analytic solution for the relativistic
disc of dust [3], a transition to a black hole was found explicitly [4]. Transi-
tions have also been found numerically for rings with a variety of equations of
state [5,6]. Such parametric transitions to a black hole can be interpreted as a
quasi-stationary collapse.

The methods we use to study parametric transitions differ from those in the
above cited papers, since we here concentrate on the behaviour of multipole
moments and on the appearance of a region of spacetime typical of metrics
close to the extreme Kerr limit. These transitions are studied for strange mat-
ter, which is considered to be a form of matter that may be astrophysically
relevant and has not been considered elsewhere for ring topologies. We restrict
our attention here to ring configurations, since there is evidence suggesting that
only rings and discs permit a transition to a black hole. We include in this paper
a comparison with the corresponding transitions of rings governed by other
equations of state.

Section 2 is devoted to a brief description of the equation of state used here
to model strange matter. In Sect. 3 we define multipole moments at infinity and
follow their progression as they tend to those of the extreme Kerr black hole.
The appearance of a “throat region” separating the “inner” from the “outer
world” is discussed in Sect. 4. In Sect. 5, we verify numerically that a particle
resting on the ring’s surface is always gravitationally bound, a condition, which
can be considered to be a minimal requirement for stability. We close with a
short summary in Sect. 6.

Throughout this paper, units are used in which the gravitational constant $G$
and speed of light $c$ are equal to one.

2 Equation of state

Strange matter is a fluid made of up (u), down (d) and strange (s) quarks. Our
equation of state (eos) to characterize strange matter is the same one described
by Gourgoulhon et al. [7], who studied the properties of axially symmetric, sta-
tionary, spheroidal strange matter configurations. Based on the MIT bag model,
we consider equal numbers of massless, non-interacting u, d, s quarks, confined
to a given volume, i.e. enclosed in a “bag”. The limits of the bag correspond in
our case to the surface of the star, such that the star is entirely composed of
strange matter. This model leads to a simple eos:

$$\epsilon = 3p + 4B,$$ \hfill (1)

where $\epsilon$ is the energy density, $p$ the pressure and $B$ the bag constant, character-
izing the quark confinement.

In the Newtonian limit, the pressure $p$ is low and negligible in comparison
to $B$, so the eos takes the form $\epsilon = \text{constant}$. Therefore, all the known New-
tonian solutions for homogeneous bodies will be found in the Newtonian limit
of the MIT bag model. Of course, a quark model of matter is not relevant in