Gen Relativ Gravit (2008) 40:269–284
DOI 10.1007/s10714-007-0547-7

RESEARCH ARTICLE

Is the evidence for dark energy secure?

Subir Sarkar

Received: 22 October 2007 / Accepted: 23 October 2007 / Published online: 13 December 2007
© Springer Science+Business Media, LLC 2007

Abstract Several kinds of astronomical observations, interpreted in the framework of the standard Friedmann–Robertson–Walker cosmology, have indicated that our universe is dominated by a Cosmological Constant. The dimming of distant Type Ia supernovae suggests that the expansion rate is accelerating, as if driven by vacuum energy, and this has been indirectly substantiated through studies of angular anisotropies in the cosmic microwave background (CMB) and of spatial correlations in the large-scale structure (LSS) of galaxies. However there is no compelling *direct* evidence yet for (the dynamical effects of) dark energy. The precision CMB data can be equally well fitted without dark energy if the spectrum of primordial density fluctuations is not quite scale-free and if the Hubble constant is lower globally than its locally measured value. The LSS data can also be satisfactorily fitted if there is a small component of hot dark matter, as would be provided by neutrinos of mass $\sim$0.5 eV. Although such an Einstein–de Sitter model cannot explain the SNe Ia Hubble diagram or the position of the "baryon acoustic oscillation" peak in the autocorrelation function of galaxies, it may be possible to do so, e.g. in an inhomogeneous Lemaitre–Tolman–Bondi cosmology where we are located in a void which is expanding faster than the average. Such alternatives may seem contrived but this must be weighed against our lack of any fundamental understanding of the inferred tiny energy scale of the dark energy. It may well be an artifact of an oversimplified cosmological model, rather than having physical reality.

Keywords Cosmic microwave background · Dark energy · Inflation · Large-scale structure

---

S. Sarkar (✉)
Rudolf Peierls Centre for Theoretical Physics,
University of Oxford, 1 Keble Road, Oxford OX1 3NP, UK
e-mail: s.sarkar@physics.ox.ac.uk

1 Introduction

Following his formulation of general relativity Einstein [29] boldly applied the theory to the universe as a whole. The first cosmological model was *static* to match the known universe, which at that time was restricted to the Milky way, and to achieve this Einstein introduced the "cosmological constant" term (for a historical perspective, see [71]). Within a decade however Slipher and Hubble demonstrated that the nebulae on the sky are in fact other "island universes" like the Milky Way and that they are mainly receeding from us—the universe is expanding. Einstein wrote to Weyl in 1933: "*If there is no quasi-static world, then away with the cosmological term*".

This however is not a matter of choice since general coordinate invariance, which Einstein's equation is based on, permits an arbitrary constant (multiplied by the metric tensor) to be added to the lhs:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \lambda_{\text{metric}} g_{\mu\nu} = \frac{-T_{\mu\nu}}{M_{\text{P}}^2}. \tag{1}$$

Here we have written Newton's constant, $G_{\text{N}} \equiv 1/8\pi\, M_{\text{P}}^2$ where $M_{\text{P}} \simeq 2.4 \times 10^{18}$ GeV is the (reduced) Planck mass in natural units ($\hbar = k_{\text{B}} = c = 1$). With the subsequent development of quantum field theory it became clear that the energy–momentum tensor on the rhs can also be freely scaled by another additive constant multiplying the metric tensor, which reflects the (Lorentz invariant) energy density of the vacuum:

$$\langle T_{\mu\nu} \rangle_{\text{fields}} = -\langle \rho \rangle_{\text{fields}} g_{\mu\nu}. \tag{2}$$

This contribution from the matter sector adds to the "bare" term from the background geometry, yielding an effective cosmological constant:

$$\Lambda = \lambda_{\text{metric}} + \frac{\langle \rho \rangle_{\text{fields}}}{M_{\text{P}}^2}, \tag{3}$$

or, correspondingly, an effective vacuum energy:

$$\rho_{\text{v}} \equiv \Lambda M_{\text{P}}^2. \tag{4}$$

Einstein *assumed* without any observational evidence that the universe is perfectly homogeneous. We know that the universe is quite isotropic about us so this is in fact likely if we are not in a special location—an assumption later dignified by Milne as the "Cosmological Principle". Then using the *maximally symmetric* Robertson–Walker metric to describe space–time

$$\mathrm{d}s^2 \equiv g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu = \mathrm{d}t^2 - a^2(t)[\mathrm{d}r^2/(1 - kr^2) + r^2 \mathrm{d}\Omega^2], \tag{5}$$