Mars and frame-dragging: study for a dedicated mission

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Abstract In this paper, we preliminarily explore the possibility of designing a dedicated satellite-based mission to measure the general relativistic gravitomagnetic Lense–Thirring effect in the gravitational field of Mars. The focus is on the systematic uncertainty induced by the multipolar expansion of the areopotential and on possible strategies to reduce it. It turns out that the major sources of bias are the Mars’ equatorial radius $R$ and the even zonal harmonics $J_{\ell}$, $\ell = 2, 4, 6, \ldots$ of the areopotential. An optimal solution, in principle, consists of using two probes at high-altitudes ($a \approx 9,500–9,600$ km) and different inclinations (one probe should fly in a nearly polar orbit), and suitably combining their nodes in order to entirely cancel out the bias due to $\delta R$. The remaining uncancelled mismodelled terms due to $\delta J_{\ell}$, $\ell = 2, 4, 6, \ldots$ would induce a bias $\lesssim 1\%$, according to the present-day MGS95J gravity model, over a wide range of admissible values of the inclinations. The Lense–Thirring out-of-plane shifts of the two probes would amount to about $10$ cm year$^{-1}$.

Keywords Experimental tests of gravitational theories · Lunar, planetary, and deep-space probes · Mars · Gravitational fields

1 Introduction

Recent claims concerning a possible detection of the general relativistic gravitomagnetic Lense–Thirring effect [1] in the gravitational field of Mars with the Mars Global Surveyor probe [2–5] may raise interest concerning such a new Solar System scenario for testing Einsteinian gravity. In view of the situation of the Gravity Probe B mission [6, 7],
which might finally reach an accuracy\textsuperscript{1} worse than the originally expected \( \lesssim 1\% \) in measuring the Schiff precessions \cite{8} of the spins of four gyroscopes carried onboard, and of the lingering controversy about the realistic accuracy reached with the terrestrial LAGEOS-LAGEOS II test of the Lense–Thirring effect \cite{9} (and reachable with the recently approved LAGEOS-like LARES satellite, to be launched at the end of \textsuperscript{2}2009 by the Italian Space Agency \cite{10}) we feel that attempts to scout new and unexplored routes may be of some value. Indeed, as many laboratories and methods as possible should be used to extensively test a fundamental interaction like gravitation and its predictions, especially when their tests are so few and their outcomes uncertain.

In this paper, we wish to fix a first stick by studying in some details the possibility of using a dedicated mission to Mars with one or more probes to measure at a reasonable level of accuracy (a few percent) the elusive Lense–Thirring effect on the longitude of the ascending node \( \Omega \) of the spacecraft’s orbital plane. We will concentrate here on one of the major source of systematic error, i.e. the multipolar expansion of the martian gravitational potential (known also as areopotential) which induces on the node a huge noise with the same temporal signature of the relativistic signal of interest (a linear rate) in order to see what are the critical issues in view of the present-day knowledge of the Martian space environment. We will not discuss here the perturbations of non-gravitational origin which depend on the shape, the instrumentation and the orbital maneuvers of such probes. They would be strongly related to possible other tasks, more consistent with planetology, which could be fruitfully assigned to such a mission in order to enhance the possibility that it may become something more than a mere, although-hopefully-interesting, speculation; in this respect the medium-long term ambitious programs of NASA to Mars may turn out to be useful also for the purpose discussed here.

2 The use of one nearly polar spacecraft

The Lense–Thirring effect consists of a small secular precession of the longitude of the ascending node\textsuperscript{3} \( \Omega \) of the orbit of a satellite moving around a central slowly rotating mass

\[
\dot{\Omega}_{LT} = \frac{2GS}{c^2a^3(1-e^2)^{3/2}},
\]

where \( G \) is the Newtonian constant of gravitation, \( S \) is the spin angular momentum of the central body, \( a \) and \( e \) are the semimajor axis and the eccentricity, respectively, of the satellite orbit. According to the latest determinations of the global properties of Mars \cite{11}, \( S = (1.92 \pm 0.01) \times 10^{32} \text{ kg m}^2 \text{s}^{-1} \) for the red planet.

The oblateness of the central body, of mass \( M \) and equatorial radius \( R \), makes the node to secularly precess as well according to \cite{12}

\textsuperscript{1} See on the WEB \url{http://einstein.stanford.edu/}.

\textsuperscript{2} See on the WEB \url{http://www.esa.int/esapub/bulletin/bulletin135/bul135f_bianchi.pdf}.

\textsuperscript{3} Also the argument of pericentre \( \omega \) of a satellite secularly precesses under the action of the gravitomagnetic force, but we will not consider here such an effect.