Thermodynamics of Lemaître–Tolman–Bondi Model

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Abstract Here we consider our universe as inhomogeneous spherically symmetric Lemaître–Tolman–Bondi Model and analyze the thermodynamics of this model of the universe. The trapping horizon is calculated and is found to coincide with the apparent horizon. The Einstein field equations are shown to be equivalent with the unified first law of thermodynamics. Finally assuming the first law of thermodynamics validity of the generalized second law of thermodynamics is examined at the apparent horizon for the perfect fluid and at the event horizon for holographic dark energy.

Keywords Thermodynamics · Inhomogeneity · Tolman–Bondi Model

1 Introduction

The discovery of Hawking radiation [1] completes the cyclic of identifying black hole (BH) as a thermodynamical object—the laws of BH physics and thermodynamical laws are equivalent. Since then there is a series of works [2–6] dealing with thermodynamical studies of the universe as thermodynamical system. Considering homogeneous and isotropic FRW model of the universe, most studies deal with validity of the generalized second law of thermodynamics (GSLT) starting from the first law when universe is bounded by the apparent horizon [7–15]. Considering various matter system and different gravity theories, it is generally found that there is a nice agreement
of the thermodynamical laws with apparent horizon as the boundary. Also it is found that first law of thermodynamics and (modified) Einstein equations are equivalent at the apparent horizon. In contrast, there are few works [16–19] dealing with thermodynamics of the universe with event horizon as the boundary. Due to existence of the event horizon, the matter here is chosen as either quintessence or exotic in nature. Here validity of GSLT put some restrictions either on geometry or on the matter itself except when the matter is in the form of holographic dark energy (HDE) [20–32], no constraint is necessary.

In the present work, we consider our universe as in homogeneous Lemaitre–Tolman–Bondi [LTB] Model. This simple inhomogeneous cosmological model agrees with current supernova and some other data [33–38]. Also very recently Clarkson and Marteens [39] give a justification for inhomogeneous model from the point view of perturbation analysis. The apparent horizon and the trapping horizon coincide for the model. We are able to show that Einstein field equations and unified first law are equivalent on the apparent horizon. Finally we determine the constrains to satisfy the GSLT on the apparent horizon for the perfect fluid and on the event horizon with matter as HDE [40].

2 Basic equations in LTB model

The metric ansatz for inhomogeneous spherically symmetric LTB space time in a co-moving frame is given by

$$dS^2 = -dt^2 + \frac{R^2}{1 + f(r)} dr^2 + R^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$  \hspace{1cm} (1)

where $R = R(r, t)$ is the (area) radius of the spherical surface and $f(r)$ ($> -1$) is the curvature scalar (classifies the space-time as bounded, marginally bounded and unbounded depending on the range of its values which are respectively $f(r) < 0$, $f(r) = 0$, $f(r) > 0$). Let us suppose that the universe is filled with perfect fluid with energy momentum tensor

$$T_{\mu \nu} = (\rho + p) u^\mu u^\nu + pg_{\mu \nu}.$$  \hspace{1cm} (2)

where $\rho$ and $p$ are respectively the matter density and pressure of the fluid and $u^\mu$ is the fluid-four velocity of the fluid with normalization $u^\mu u^\nu = -1$. By introducing the mass function $F(r, t)$ [41–44] (related to the mass contained within the co-moving radius $r$) as

$$F(r, t) = R(\dot{R}^2 - f(r))$$  \hspace{1cm} (3)

the Einstein equations can be written as

$$8\pi G \rho = \frac{F'(r, t)}{R^2 R'}, \quad 8\pi G p = -\frac{\dot{F}(r, t)}{R^2 R}$$  \hspace{1cm} (4)