Naked singularity formation in $f(\mathcal{R})$ gravity

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Received: 28 November 2010 / Accepted: 13 June 2011 / Published online: 2 July 2011
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Abstract We study the gravitational collapse of a star with barotropic equation of state $p = w\rho$ in the context of $f(\mathcal{R})$ theories of gravity. Utilizing the metric formalism, we rewrite the field equations as those of Brans-Dicke theory with vanishing coupling parameter. By choosing the functionality of Ricci scalar as $f(\mathcal{R}) = \alpha\mathcal{R}^m$, we show that for an appropriate initial value of the energy density, if $\alpha$ and $m$ satisfy certain conditions, the resulting singularity would be naked, violating the cosmic censorship conjecture. These conditions are the ratio of the mass function to the area radius of the collapsing ball, negativity of the effective pressure, and the time behavior of the Kretschmann scalar. Also, as long as parameter $\alpha$ obeys certain conditions, the satisfaction of the weak energy condition is guaranteed by the collapsing configuration.

Keywords Naked singularity · $f(\mathcal{R})$ gravity

1 Introduction

Einstein’s General theory of relativity is the classical theory of one of the four fundamental forces, gravity, which is the weakest but most dominant force of nature governing phenomena at large scales, and is described by a mathematically well-founded and elegant structure i.e., differential geometry of curved spacetime. The Einstein’s field equations, a system of non-linear partial differential equations, relate
the geometric property of spacetime to the four-momentum (energy density and linear momentum) of matter fields leading to precise predictions that have received considerable experimental confirmations with high accuracy such as solar system tests (see [1] and references therein). One of the most engrossing but open debates in general relativity is that of the final fate of gravitational collapse with possibility of the existence of spacetime singularities, the ultra-strong gravity regions where the densities and spacetime curvatures blow up, leading to a spacetime which is geodesically incomplete [2] and the structure of any classical theory of fields is vanquished. A star with a mass many times than that of the Sun would undergo a continual gravitational collapse due to its self-gravity without achieving an equilibrium state in contrast to a neutron star or a white dwarf. Then, according to singularity theorems established by Hawking and Penrose [3–5] a singularity is reached as the collapse endstate. Such a singularity may be a black hole hidden from external observers by an event horizon or visible to the outside Universe (naked singularity). In the latter collapse procedure, the information on super-dense regions can be transported via suitable non-spacelike trajectories to a distant observer. Although the occurrence of a spacetime singularity as the final outcome of a collapse scenario is proved by the singularity theorems, they do not specify the nature of such a singularity. The cosmic censorship conjecture first articulated by Penrose [6] states that a black hole is always formed in complete gravitational collapse of reasonable matter fields, or a physically reasonable spacetime contains no naked singularities. However, up to now many exact solutions of Einstein’s field equations describing singularities, not hidden behind an event horizon of spacetime, are known. A remarkable study is the one by Shapiro and Teukolsky [7,8], who showed numerically that gravitational collapse of a spheroidal dust may end in a naked singularity. Also many exact solutions of Einstein’s field equations with a variety of field-sources admitting naked singularities have been surveyed. The examples studied so far include gravitational collapse of a pressure-less matter [9–14], radiation [15–22], perfect fluids [23–25], imperfect fluids [26,27] and null strange quark fluids [28–30]. Beside general theory of relativity, there exist alternative theories of gravity explaining gravitational phenomena. Such theories have been studied for a long time [31–35]. From the theoretical standpoint there has been many attempts to correct the Einstein-Hilbert action in order to renormalize general relativity to build a quantum theory of gravity or at least some effective action (including the low-energy limit of string theories), or to quantize the scalar fields in curved spacetimes [36]. From the observational point of view, the discovery of current acceleration of the Universe using CMB Ia supernova [37–45] suggests that such acceleration may be explained within the framework of general relativity by assuming that 76% of energy content of the Universe is filled with a mysterious form of dark energy with equation of state $p \sim -\rho$ (where $\rho$ and $p$ are the energy density and pressure of the cosmic fluid, respectively). Another possibility is to include a cosmological constant $\Lambda$ of a very small magnitude in Einstein’s field equation, but encounters such difficulties as the well-known cosmological constant problem and the coincidence problem. Another alteration is $f(R)$ theories of gravity [46–50] where the Ricci scalar in the Einstein-Hilbert Lagrangian is replaced by a general function of it, providing alternative gravitational models for dark energy since the explanation of the cosmic acceleration comes back to the fact that we do not understand gravity at large scales. Such theories can describe the transition