MATHEMATICAL DESCRIPTION OF TOUGHENING OF PASSENGER VEHICLE WINDOW GLASS

R. I. Makarov¹ and E. V. Suvorov²


Mathematical models describing the effect of the toughening process on passenger vehicle window glass, specifically, the character of fracturing during tests, are developed. Linear regression models adequately describe the maximum number and length of the fragments obtained during the tests. It is shown that the toughening regime affects the character of the fracturing of the glass.

Key words: control of glass toughening, glass quality, modeling of the process.

Continuous-flow production of toughened glass consists of successive manufacturing stages: cutting and working glass blanks, toughening the glass, packaging the finished product, and controlling unconforming product. Many factors determine product quality. Quality management systems which are in continually operation during the manufacturing process to maintain product quality at a high level are developed to ensure high quality. Simulation is used as a tool to investigate the manufacturing processes and generate recommendations for making control decisions [1].

The concepts of key product characteristics — critical and major — are linked with quality.

According to GOST 5727–88 “Safety Glass for Ground Transportation” the mechanical strength and character of the fracture of toughened glass are critical characteristics. Deviations of bent articles from the prescribed shape, absence of chipping on the open ends, light transmission, and optical distortions are major characteristics [2].

The toughening process occurs in a continuous-flow horizontal furnace. Glass blanks are heated in a four-chamber tunnel furnace, after which they are pressed to a prescribed shape. Next, glass with a fixed shape is toughened by rapid cooling in a stream of water. After toughening the glass is allowed to cool slowly.

The toughening process is a complicated, weakly structured, poorly formalized object. Many controlled and uncontrolled characteristics and their inter-relations influence the result of the toughening process simultaneously. As a result the conventional methods of performing studies are of little use. Cognitive structuring methods were used to investigate the process [3]. Production specialists who know the manufacturing process for toughened glass and have experience in controlling this process (production chief, production engineer, experienced hardener) were called upon to investigate the process. The results of queries were used to construct a cognitive model which interprets the specialists’ opinions and knowledge. Subsequently, the cognitive model was used to choose a structure for the regression models being developed, and the factors and process regimes affecting the character of the fracture were determined.

In the tests performed to determine the character of the fracture there must be at least 40 and no more than 400 fragments in a 50 × 50 mm square. The length of the fragments must not exceed 75 mm (GOST 5727–88). Four samples are tested. The number of fragments is calculated for glass in zones with the coarsest and finest fracturing, and the minimum and maximum number of fragments and the maximum length of the fragments are recorded.

A correlation analysis of the test results was performed to choose a representative impulse according to the character of the fracture. The correlation coefficients between the indicators of the character of the fractures of toughened glass are presented in Table 1.

The analysis revealed a close statistical correlation between the maximum and minimum number of fragments in the objects tested; the partial correlation coefficient is 0.78. The direct relation agrees with the influence of the toughening process on the mechanical strength of the glass. Over-tempering of glass increases and undertempering decreases the number of fragments formed during the tests. It is sufficient to construct a single model, for example, for the maximum number of fragments, to evaluate the quality of the toughening.

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In the tests there is no correlation between the minimum number of fragments and their length. The partial correlation coefficient is $-0.17$; this is statistically insignificant at significance level 0.05.

An inverse statistical correlation exists between the maximum number of fragments and the length. This is indicated by the negative sign of the partial correlation coefficient, equal to $-0.43$. This can be explained by the fact that overtempered glass fractures during the tests into a large number of small fragments. The correlation between the number of fragments and the length is not strong; this makes it possible to construct independent models describing the character of the fractures occurring during the tests — the number and length of the fragments.

The models were constructed for $760 \times 600$ mm colorless side passenger vehicle windows and $686 \times 526$ mm rear passenger-side window. The analysis was performed on a sample of 35 observations. The equation for the maximum number of fragments $y_1$ has the form

$$y_1 = 442.3 - 26.6x_{13} - 316.4x_{18} - 8.0x_{19},$$  \hspace{1cm} (1)

where $x_{13}$ is the number of streams; $x_{18}$ is the interval 2 left-hand side; $x_{19}$ is the Poisson height, mm; and, 442.3, 26.6, 316.4, and 8.0 are coefficients in the regression equation.

The coefficient of determination of the equation is $R^2 = 86.6\%$, which is statistically significant with significance level 0.05. The relative error of the model is 7.7%; this indicates that accuracy of the model is acceptable.

The coefficient of elasticity characterizes the degree to which factor variables affect the maximum number of fragments.

### Computed Coefficients of Elasticity of the Model (1)

<table>
<thead>
<tr>
<th>Factor variable</th>
<th>Coefficient of Elasticity</th>
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</thead>
<tbody>
<tr>
<td>$x_{13}$</td>
<td>$-0.21$</td>
</tr>
<tr>
<td>$x_{18}$</td>
<td>$-0.68$</td>
</tr>
<tr>
<td>$x_{19}$</td>
<td>$-0.62$</td>
</tr>
</tbody>
</table>

The minus signs of the coefficients of elasticity attest an inverse relation between the resulting and factor variables, i.e., as the factor variables increase, the resulting variable decreases.

Variations of the factors $x_{18}$ and $x_{19}$ have the greatest effect on the maximum number of fragments in the tests performed on toughened glass. A 1% change in the value of any of the indicated factors induces a change (%) in the dependent variable $y_1$ by the value of the coefficient of elasticity corresponding to this factor.

The $\beta$ coefficients were calculated to evaluate the effect of the variability of the maximum number of fragments in the tests.

### Computed Coefficients of Elasticity of the Model (2)

<table>
<thead>
<tr>
<th>Factor variable</th>
<th>Coefficient of elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{13}$</td>
<td>$3.00$</td>
</tr>
<tr>
<td>$x_{13}$</td>
<td>$0.16$</td>
</tr>
<tr>
<td>$x_{19}$</td>
<td>$1.11$</td>
</tr>
<tr>
<td>$x_{19}$</td>
<td>$0.41$</td>
</tr>
</tbody>
</table>

The variability of the Poisson height in the process of producing toughened glass engenders the greatest variability of the dependent variable $y_1$. A change in $x_{19}$ by the amount of its variance, i.e., by 4.7 mm, engenders a change in the maximum number of fragments obtained in the tests by a fraction equal to the variance of the $\beta$ coefficient, i.e., by $53.2\beta = 37.2$ pieces. For this reason, the Poisson height of the setup ($x_{19}$) must be stabilized during press retooling in order to stabilize the number of fragments obtained during the tests.

The following regression equation adequately describes the maximum length of the fragments $y_2$ (mm):

$$y_2 = -196.5 + 6.0x_{13} + 154.8x_{18} + 1.6x_{19} + 0.25x_6,$$  \hspace{1cm} (2)

where $x_6$ is the dome temperature in zone 2 of chamber 2 of the tempering furnace, °C, and 196.5, 6.0, 154.8, and 0.25 are the coefficients of the regression equation.

The coefficient of determination of the equation is $R^2 = 83.3\%$, which is statistically significant with significance level 0.05. The relative error of the model is 6.6%, indicating that the accuracy of the model developed is adequate.

The degree to which the factor variables affect the length of the fragments obtained in the tests is characterized by the coefficient of elasticity.