The Spatial Structure of a Current Layer in an MGD Channel

E. N. Vasil’ev and D. A. Nesterov

Institute of Computational Modeling, Siberian Division, Russian Academy of Sciences, Krasnoyarsk, 660036 Russia

Abstract—The numerical solution of an unsteady-state three-dimensional set of equations of radiation gas dynamics is used to investigate the process of formation of a current layer in an MGD channel. It is found that the structure of current layer and the integral characteristics of interaction at low and high pressures are different because of the different patterns of radiation (volume and surface, respectively). The processes of flow past the discharge region and the division of this region into several current-conducting channels because of the development of Rayleigh–Taylor instability result in a decrease in the efficiency of MGD interaction.

INTRODUCTION

One of the main factors defining the efficiency of magnetogasdynamic (MGD) interaction is the electrical conductivity of the medium. In MGD flows with a self-sustaining current layer (T-layer), the electrical conductivity is supported by the thermal ionization of gas in the local plasma zone of flow, which interacts with the external magnetic field and nonconducting gas. The numerical simulation of the process in a two-dimensional formulation with a constant value of the load factor revealed that a mode of nonconducting gas flow past the current layer sets in under conditions of MGD interaction; in so doing, a vortex wake is formed downstream [1]. In addition, Rayleigh–Taylor instability (RTI) may develop on the upstream boundary of the T-layer where media of different densities are in contact and the force of gasdynamic pressure is effective; this instability leads to the division of the discharge region into two or several current-conducting channels. The characteristics of the MGD process depend on the current layer structure (the size and the distribution of electrical conductivity); therefore, its variation because of the vortex separation and division of the discharge region leads to significant fluctuations of the values of characteristics in time. The inclusion of the factors of flow and development of RTI in a two-dimensional model produced a significant difference in the integral characteristics of the process compared to those obtained in a one-dimensional approximation. More complete information about the pattern of flow in an MGD channel may be obtained as a result of computational simulation in a three-dimensional formulation. It is the objective of this study to determine the spatial form of current layer and the effect of this form on the structure of flow and integral characteristics of the process of MGD interaction for different values of pressure in the channel.

FORMULATION OF THE PROBLEM AND METHOD OF SOLUTION

We will treat the flow of a nonviscous radiating gas in a channel of constant square cross section with solid electrodes closed on an external ohmic load (Fig. 1). The nonconducting flow includes a local plasma region interacting with a transverse magnetic field. In so doing, induced electric current flows in the

Fig. 1. Diagrammatic view of the process in the channel of an MGD generator with a T-layer.
“current layer–electrodes–load” circuit. The computational model of the process is based on the solution of equations of gas dynamics, Maxwell equations, and equation of radiative transfer.

**Set of Equations**

The motion of gas is described using a set of Euler equations

\[
\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{F}}{\partial y} + \frac{\partial \mathbf{G}}{\partial z} = \mathbf{S},
\]

(1)

\[
\mathbf{U} = \begin{bmatrix} \rho u \\ \rho v \\ \rho w \\ E_i \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ (E_i + p)u + q_x \end{bmatrix},
\]

\[
\mathbf{F} = \begin{bmatrix} \rho v \\ \rho vv u \\ \rho v^2 + p \\ \rho vw \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} \rho w \\ \rho wu \\ \rho vw \\ (E_i + p)w + q_y \end{bmatrix},
\]

(2)

\[
\mathbf{S} = \begin{bmatrix} f_x \\ f_y \\ f_z \\ Q \end{bmatrix},
\]

(3)

\[
E_i = \rho \left( \frac{u^2 + v^2 + w^2}{2} + e \right),
\]

\[
Q = Q_f - Q_R + f_x u + f_y v + f_z w,
\]

and

\[
q_x = -\lambda_x \frac{\partial T}{\partial x}, \quad q_y = -\lambda_y \frac{\partial T}{\partial y}, \quad q_z = -\lambda_z \frac{\partial T}{\partial z},
\]

(4)

complemented by the equations of state

\[
p = p(\rho, e), \quad T = T(\rho, e),
\]

(5)

where \( p \), \( T \), and \( e \) denote the gas density, temperature, and pressure, respectively; \( u \), \( v \), and \( w \) are components of the gas velocity vector \( \mathbf{v} \); \( E_i \) is the total energy per unit volume of gas; \( e \) is the internal energy per unit mass of gas; \( f_x, f_y \), and \( f_z \) are components of the vector of force \( \mathbf{f} \) acting on the gas along the \( x \), \( y \), and \( z \) axes, respectively; \( Q_f \) is the volumetric power of Joule dissipation, and \( Q_R \) is the variation of energy due to radiative heat transfer; and \( \lambda \) is the thermal conductivity coefficient of gas.

The initial conditions for Eqs. (1)–(5) are preassigned in the form of distributions of temperature, pressure, and velocity,

\[
T(x, y, z, t)|_{t=0} = T_0(x, y, z),
\]

\[
p(x, y, z, t)|_{t=0} = p_0(x, y, z),
\]

\[
u(x, y, z, t)|_{t=0} = u_0(x, y, z),
\]

\[
u(x, y, z, t)|_{t=0} = 0, \quad w(x, y, z, t)|_{t=0} = 0.
\]

The boundary conditions are defined by the parameters of supersonic flow at the channel inlet,

\[
T(x, y, z, t)|_{x=L} = T_1(y, z, t),
\]

\[
p(x, y, z, t)|_{x=L} = p_1(y, z, t),
\]

\[
u(x, y, z, t)|_{x=L} = u_1(y, z, t),
\]

\[
u(x, y, z, t)|_{x=L} = 0, \quad w(x, y, z, t)|_{x=L} = 0,
\]

and “soft” boundary conditions, i.e., equality to zero of the derivatives of the sought quantities, are preassigned at the outlet,

\[
\frac{\partial T(x, y, z, t)}{\partial x} \bigg|_{x=L} = 0, \quad \frac{\partial p(x, y, z, t)}{\partial x} \bigg|_{x=L} = 0,
\]

\[
\frac{\partial \mathbf{v}(x, y, z, t)}{\partial x} \bigg|_{x=L} = 0.
\]

Impermiability conditions are preassigned on the side walls of the channel.

The following equations are solved in order to calculate the volumetric variation of energy \( Q_R \) due to radiative transfer:

\[
\mathbf{W} = \int d\Omega \mathbf{a} I_v d\Omega,
\]

(7)

\[
Q_R = \text{div} (\mathbf{W}),
\]

(8)

where \( I_v \) is the intensity of energy of radiation of frequency \( v \), \( I_{vp} \) is the intensity of equilibrium radiation, \( k_v(T, \rho, p) \) is the radiation absorption coefficient, \( \mathbf{a} \) is the unit vector which defines the direction of radiation for angle \( d\Omega \), and \( \mathbf{W} \) is the vector of radiation energy flux.