Exponential Potentials and Attractor Solution of Dilatonic Cosmology

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Abstract We present the scalar-tensor gravitational theory with an exponential potential in which Pauli metric is regarded as the physical space-time metric. We show that it is essentially equivalent to coupled quintessence (CQ) model. However for baryotropic fluid being radiation there are in fact no coupling between dilatonic scalar field and radiation. We present the critical points for baryotropic fluid and investigate the properties of critical points when the baryotropic matter is specified to ordinary matter. It is possible for all the critical points to be attractors as long as the parameters $\lambda$ and $\beta$ satisfy certain conditions. To demonstrate the attractor behaviors of these critical points, We numerically plot the phase plane for each critical point. Finally with the bound on $\beta$ from the observation and the fact that our universe is undergoing an accelerating expansion, we conclude that present accelerating expansion is not the eventual stage of universe. Moreover, we numerically describe the evolution of the density parameters $\Omega$ and the decelerating factor $q$, and computer the present values of some cosmological parameters, which are consistent with current observational data.

Keywords Exponential potential · Scalar-tensor · Dark energy · Critical point · Attractor

1 Introduction

The evidences for the existence of dark energy have been growing in past few years. Recently the WMAP three result [1] has dramatically shrunk the allowed volume in the parameter space. It shows that in a spatial flat universe the combination of WMAP and the
Supernova legacy survey (SNLS) data yields a significant constraint on the equation of state of the dark energy \( w = -0.97^{+0.07}_{-0.09} \). Though the \( \Lambda \)CDM model is still an excellent fit to the WMAP data, it still does not exclude other alternative model for the candidate of dark energy. Moreover the well-known fine-tuning and coincidence problems [2, 3] are yet unsolved in cosmological constant model. This motivates a wide range of theoretic studies to explain the observation: such as the conventional “quintessence” scalar field [4–8]; the k-essence field [9–16]; quintom model [17–23]; holograph dark energy [24–29]; Born–Infeld scalar or vector field theory [30–37]; phantom model [38–47] and so on. Additionally, some authors attempt to modify the conventional gravitational theory instead of involving the exotic matter [48–54].

In past several years the idea that dilaton field of the scalar-tensor gravitational theory as the dark energy has been proposed and discussed [55–63]. In our previous paper [64], we have considered a dilatonic dark energy model, based on Weyl-scaled induced gravitational theory. In that paper, we find that when the dilaton field is not gravitational clustered at small scales, the effort of dilaton can not change the evolutionary law of baryon density perturbation, and the density perturbation can grow from \( z \sim 10^3 \) to \( z \sim 5 \), which guarantees the structure formation. When dilaton energy is very small compared the matter energy, potential energy of dilaton field can be neglected. In this case, the solution of cosmological scale \( a \) has been found [65, 66]. In recent work [67], we consider the dilaton field with positive kinetic energy and with negative kinetic energy and find that the coupled term between matter and dilaton can’t affect the existence of attractor solutions. In this paper we will study the attractor properties of the dynamical system. The potential we choose for investigation is the exponential form for its important role in higher-order or higher-dimensional gravitational theories, string theories and Kaluza–Klein model (see the references in Ref. [68]). Though the possible cosmological roles of exponential potential have been investigated elsewhere [69–79], here we will investigate its cosmological implies in our dilatonic cosmology. With the constraint from the observation we conclude that the present accelerating expansion is not the eventual stage of universe.

2 Theoretical Model from Scalar-Tensor Gravitational Theory

The action of Jordan–Brans–Dicke theory is:

\[
S = \int d^4x \sqrt{-\gamma} \left[ \phi \tilde{\mathcal{R}} - \omega \gamma_{\mu \nu} \frac{\partial_{\mu} \phi \partial_{\nu} \phi}{\phi} - \Lambda(\phi) - \tilde{\mathcal{L}}_{\text{fluid}}(\psi) \right]
\]  

where \( \tilde{\mathcal{L}}_{\text{fluid}} \) is the Lagrangian density of cosmic fluid, \( \gamma \) is the determinant of the Jordan metric tensor \( \gamma_{\mu \nu} \), \( \omega \) is the dimensionless coupling parameter, \( \tilde{\mathcal{R}} \) is the contracted \( \tilde{\mathcal{R}}_{\mu \nu} \). The metric sign convention is \((- , + , + , + )\). The quantity \( \Lambda(\phi) \) is a nontrivial potential of \( \phi \) field. When \( \Lambda(\phi) \neq 0 \) the (1) describes the induced gravity. \( \tilde{\rho} \) and \( \tilde{p} \) is respectively the density and pressure of cosmic fluid. However it is often useful to write the action in terms of the conformally related Einstein metric. We introduce the dilaton field \( \sigma \) and conformal transformation as follows:

\[
\phi = \frac{1}{2\kappa^2} e^{\sigma},
\]

\[
\gamma_{\mu \nu} = e^{-\alpha \sigma} g_{\mu \nu}
\]