A heat-conduction problem is formulated for laminated plates and shells with a heat-conducting layer and debonding between laminas. The approach consists in analyzing how the layer thickness changes in the process of debonding of laminas and deformation of plates and shells. The three-dimensional thermoelastic and heat-conduction equations are expanded into polynomial Legendre series in thickness. The first-order, Timoshenko’s, and Kirchhoff–Love equations are examined. A numerical example of laminated shells with a heat-conducting layer is considered.

Keywords: laminated plates and shells, heat conductivity, heat-conducting layer, debonding of laminas, polynomial Legendre series, numerical example

1.3-D Formulation. Let an elastic homogeneous anisotropic laminated shell of arbitrary geometry consist of \( Q \) layers with \( 2h^q \) thickness. Here and henceforth, all the parameters related to layers are marked with superscripts in brackets. The same parameters related to the whole shell have no subscripts in brackets.

We consider the possibility of debonding between laminas. There is a heat-conducting medium in the gap \( h_0(x) \) between laminas in the debonding area. The medium does not resist the deformation of laminas, and the heat exchange between laminas is due to the thermal conductivity of the medium. We assume that \( h_0 \) is commensurable with the displacements of laminas and that these displacements are small.

The thermodynamic state of the system, including the laminas and the heat-conducting medium, is defined by the following parameters: \( h_0(x), \varepsilon^{(q)}_{ij}(x), \) and \( u^{(q)}_i(x) \) are the components of the stress and strain tensors and displacement vector, and \( \Theta^{(q)}(x), \chi^{(q)}(x), \Theta^*(x), \) and \( \chi^*(x) \) are the temperature and specific strength of internal heat sources in the bodies and the medium, respectively. In this case, the following relations hold [1, 4]:

\[
\partial_j \sigma^{(q)}_{ij} + b^{(q)}_j = 0, \quad \varepsilon^{(q)}_{ij} = \frac{1}{2} (\partial_i u^{(q)}_j + \partial_j u^{(q)}_i), \quad \sigma^{(q)}_{ij} = c^{(q)}_{ijkl} \varepsilon^{(q)}_{kl} + \beta^{(q)}_{ij} h^{(q)},
\]

(1)

where \( \partial_j = \partial / \partial x_j \) are partial derivatives with respect to the space variables \( x_j \); and \( c^{(q)}_{ijkl} \) and \( \beta^{(q)}_{ij} \) are the elastic modulus and the coefficients of linear thermal expansion. In the isotropic case,

\[
c^{(q)}_{ijkl} = \lambda^{(q)} \delta_{ij} \delta_{kl} + \mu^{(q)} (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}), \quad \beta^{(q)}_{ij} = (\mu^{(q)} + 2\lambda^{(q)}) \kappa \delta_{ij},
\]

where \( \lambda \) and \( \mu \) are the Lamé constants, \( \alpha^{(q)} \) are the coefficients of linear thermal expansion, and the summation convention for repeated indices is adopted.

The differential equilibrium equations for the displacement components may be presented in the form...
with $A_{ij}^{(q)} = \epsilon_{ijkl} \partial_k \partial_l$, $A_i^{(q)} = \beta_{ij} \partial_j$, and $A_{ij}^{(q)} = \mu^2 \delta_{ij} \partial_k \partial_l + (\lambda^{(q)} + \mu^{(q)}) \partial_i \partial_j$, $A_i^{(q)} = (\mu^{(q)} + 3\lambda^{(q)}) \chi^{(q)} \partial_i$ in the anisotropic and isotropic cases, respectively.

Boundary conditions for displacements and traction on the parts $\partial V_p^{(q)}$ and $\partial V_u^{(q)}$ have the form

$$p_i^{(q)} = g_{ij}^{(q)} n_j = q_i^{(q)}, \quad \forall x \in \partial V_p^{(q)}, \quad u_i^{(q)} = q_i^{(q)}, \quad \forall x \in \partial V_u^{(q)}, \quad \forall x \in V^{(q)}. \quad (3)$$

In the debonding areas $\partial V_e^{(q)}$, the boundary conditions at the crack edges have the form of inequalities [2, 3, 6]:

$$\Delta u_n^{(q)} = u_n^{(q)} - u_n^{(q+1)} \geq h_0^{(q)}, \quad g_n^{(q)} \geq 0, \quad \left( \Delta u_n^{(q)} - h_0^{(q)} \right) g_n^{(q)} = 0,$$

$$p^{(q)} = p^{(q+1)} = q^{(q)}, \quad n^{(q)} = -n^{(q+1)}, \quad \forall x \in \partial V_e^{(q)}, \quad \partial V_e^{(q)} = \partial V_{e^{(q)}} \cap \partial V_{e^{(q+1)}}, \quad (4)$$

where $g_n^{(q)}$, $\Delta u_n^{(q)} = u_n^{(q)} - u_n^{(q+1)}$, $q^{(q)}$, and $\Delta u_t^{(q)} = u_t^{(q)} - u_t^{(q+1)}$ are the normal and tangential components of the contact force vector and the displacement discontinuity vector, respectively; and $k^{\tau(q)}$ and $\lambda^{\tau(q)}$ are coefficients that depend upon the properties of the contact surfaces.

The linear heat-conduction equations have the form

$$\lambda_{ij}^{(q)} \partial_i \partial_j \theta^{(q)} - \chi^{(q)} = 0, \quad \forall x \in V^{(q)}, \quad (5)$$

where $\lambda_{ij}^{(q)}$ are the coefficients of thermal conductivity. In the isotropic case, we have $\lambda_{ij}^{(q)} = \delta_{ij} \lambda^{(q)}$. The boundary conditions for temperature and heat flux on the parts $\partial V_\theta^{(q)}$ and $\partial V_\tau^{(q)}$ have the form

$$\theta^{(q)} = \theta_{b}^{(q)}, \quad \forall x \in \partial V_\theta^{(q)}, \quad q^{(q)} = q_{b}^{(q)}, \quad \forall x \in V^{(q)}. \quad (6)$$

The temperature distribution within the heat-conducting medium is described by the heat-conduction equations

$$\lambda_{ij}^{\theta} \partial_i \partial_j \theta^{\theta} - \chi^{\theta} = 0, \quad \forall x \in V^{\theta}. \quad (7)$$

The boundary conditions on the lateral sides of the heat-conducting medium have the form

$$\lambda_{ij}^{\theta} \partial_i \partial_j \theta^{\theta} + \beta_{ij} \left( \theta^{\theta} - \theta_{b}^{\theta} \right) = 0. \quad (8)$$

The heat-conduction conditions for the heat-conducting medium have the form

$$\theta^{\theta} = \theta^{(q)}, \quad \lambda_{ij}^{\theta} \partial_i \theta^{\theta} = \lambda_{ij}^{\theta} \partial_i \theta^{(q)}, \quad \forall x \in \partial V_{\theta^{(q)}}. \quad (9)$$

In the area of close mechanical contact, the thermal conditions transform into the form

$$q_0 = \alpha_{e} \left( \theta^{(q)} - \theta_{b}^{(q)} \right), \quad \forall x \in \partial V_{e^{(q)}}, \quad (10)$$

where $q_0$ is the heat flux across the close mechanical contact area, and $\alpha_{e}$ is the contact heat conductivity.

The analysis of the problem encounters mathematical difficulties caused by the dimension and non-linearity of the problem. The problem can be partially simplified by considering thin bodies. In this case, we can reduce the dimension of the problem.