DYNAMICS OF ELASTIC BODIES, SOLID PARTICLES, AND FLUID PARCELS IN A COMpressible Viscous FLUID (REVIEW)

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The results of linearization of the basic equations describing a compressible viscous fluid in which low-amplitude oscillations occur or solids move or that interacts with elastic bodies in which small perturbations propagate are discussed. The general solutions of the linearized equations are presented. The results of studying wave processes in hydroelastic systems using the three-dimensional linearized theory of finite deformations and theory of compressible viscous fluid are discussed. The results of studying the propagation of acoustic waves of various types in waveguides with plane and circular cylindrical interfaces between elastic and liquid media and the influence of large (finite) initial deformations, viscosity and compressibility of the fluid on acoustic waves are presented. Studies of the motion of objects in compressible ideal and viscous fluids under the action of radiation forces due to the acoustic field are reviewed. The emphasis is placed on the studies that use a method involving the solution of hydrodynamic problems for a compressible fluid with solid particles and the evaluation of the forces acting on these particles. The radiation force is determined as the constant component of the hydrodynamic force. The numerical results are presented in the form of plots, which are then analyzed.

Keywords: three-dimensional linearized theory, viscous compressible fluid, compressible and incompressible elastic body, prestress, radiation force, acoustic field, particle, harmonic wave

Introduction. The dynamics of a fluid interacting with rigid and deformable bodies is one of the fundamental and classical problems of mechanics, physics, and applied mathematics. The results obtained in studying this problem are of applied importance for various problems of natural sciences and engineering, including newest technologies. So far, the dynamic behavior of rigid and elastic bodies in a viscous fluid has mainly been studied assuming that the fluid is incompressible and viscous. There are few publications where the fluid is considered to be viscous and compressible (see [31, 76, 78, 91, 95, 171] for reviews of such studies). An analysis of the results shows that, despite the importance of the dynamic problem, the low-amplitude vibrations of and the motion/interaction of rigid and elastic bodies in/with a compressible viscous fluid have been studied inadequately.

The present paper is a review of the results obtained in studying, using the three-dimensional linearized theory, the motion and interaction of solid particles in a fluid and the propagation of low-amplitude waves in elastic bodies interacting with a compressible viscous fluid.

Section 1 deals with the linearized equations for a resting compressible viscous fluid undergoing nonstationary and harmonic low-amplitude motions (oscillations). The exact expression for the total derivative of the velocity vector will be used. The linearized theory for a resting compressible viscous fluid will be shown to be analogous to a specific rheological model of a solid. The general solutions of the basic equations of the linearized theory for a compressible viscous fluid will be expressed in terms of scalar and vector potentials. The equations from which these potentials are derived will be presented as well. It will be...
shown that by passing to the limit, these general solutions can be reduced to general solutions for simpler fluid models (incompressible viscous fluid; compressible or incompressible ideal fluid).

The development of engineering and industry requires using theories that more adequately describe the properties of real elastic and liquid media. In this connection, it is important to use prestressed body and compressible viscous fluid models to study wave processes. Using such an approach is reasonable because neglecting the prestresses of the body and the viscosity of the fluid changes the structure of the differential equations so much that many real phenomena cannot be studied even qualitatively and the qualitative results obtained with approximate models do not meet the ever-toughening accuracy requirements. It is, therefore, of theoretical and applied interest to study wave processes in hydroelastic systems using the equations of the three-dimensional linearized theory of elasticity with the assumption of finite deformation for the body and the three-dimensional linearized Navier–Stokes equations for the compressible viscous fluid. These issues are covered in Sec. 2.

Section 2 addresses the results of studying wave processes in hydroelastic waveguides with plane and curved interfaces. We will discuss statements of and problem-solving methods for the basic classes of aero-hydroelastic problems for compressible and incompressible elastic bodies subjected to large (finite) initial deformation and interacting with a compressible viscous fluid. We will analyze the graphs of numerical solutions of the dispersion equations for some specific problems. They show how the viscosity of the compressible fluid and the prestresses of the body affect the phase velocities, damping factors, and dispersion of normal waves.

Section 3 analyzes the results of studying the interaction of an acoustic wave with rigid and elastic particles in a compressible (viscous or ideal) fluid, the measure of this interaction being radiation (period-average) forces. Radiation forces are caused by the time-independent radiation stresses occurring in an acoustic field. Unlike pressure, they are tensor quantities [162, 163]; hence, the components of the force vector acting on a unit area of the body can be determined as the inner product of the stress tensor and the unit normal vector to the area. This is not so with scalar pressure. The acoustic literature, however, uses the term “radiation pressure” for this vector quantity [48, 137, 145]. A change in the time-average momentum flux within some volume of a fluid is responsible for the occurrence of radiation stresses during the propagation of an acoustic wave (there were controversies in the literature over the momentum and flux of momentum of various waves [48, 137, 145]). Such changes are due to second-order effects such as scattering of sound by an obstacle, absorption of sound by a propagation medium, etc. Therefore, radiation pressure is a quadratic function of the acoustic-field variables. In Lagrangian coordinates, acoustic radiation pressure is the time-average acoustic pressure on the surface of an obstacle.

In this connection, linear approximation is insufficient to calculate the acoustic pressure exerted by a harmonic wave because the pressure is a periodic function of time in this approximation [136] and its average over the wave period is equal to zero. Therefore, to determine the acoustic pressure in a fluid, it is necessary to take into account the second-order effects due to the inharmonicity of the wave profile near the obstacle. Radiation pressure strongly affects the motion of a solid particle in the fluid and imparts a unidirectional displacement to it. If there are several particles in a fluid, the interference of the incident and reflected (from the particles) waves creates a complex acoustic field in which radiation forces different in magnitude and direction act on the particles, causing their relative drift. This circumstance is widely used to intensify many acoustically assisted processes [147, 152].

There are two types of acoustic radiation pressure [48, 49, 137, 141]: Rayleigh and Langevin, or Langevin–Brillouin. Rayleigh pressure occurs when waves propagate without interaction between the acoustic field and the unperturbed medium. These cases are characteristic of closed volumes where the mass of the oscillating medium remains constant. An example is the case, studied by Rayleigh, of plane standing waves between two fixed flat solid surfaces. The expression for Rayleigh acoustic pressure depends on a coefficient that describes the nonlinear properties of the medium.

Langevin pressure occurs when an acoustic field interacts with an unperturbed medium that affects the time-average pressure. An example is waves damped at infinity. Langevin acoustic pressure in one-dimensional acoustic fields was studied in [48, 144, 161, 179, etc.].

Here we will discuss studies on the interaction of acoustic waves with rigid and elastic particles in a bounded or unbounded viscous or ideal fluid conducted at the S. P. Timoshenko Institute of Mechanics.

1. Basic Equations of the Linearized Theory of Compressible Viscous Fluid. To adequately describe the dynamics of rigid and elastic bodies in a fluid, it is necessary to devise suitable fluid models, because rigid or elastic solid models are predetermined by a specific problem statement. The compressible viscous fluid model is the most general of the classical fluid models, because it combines the property of compressibility, which allows describing the propagation of waves in a compressible ideal fluid, and the property of viscosity, which allows describing the damping of dynamical processes in an