UNSTEADY RADIATIVE–CONVECTIVE HEAT TRANSFER
IN A HIGH-TEMPERATURE GAS–PARTICLE FLOW PAST
A SEMI-TRANSPARENT PLATE

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Numerical simulations of unsteady radiative-convective heat transfer in a turbulent flow of a mixture of gases and solid particles past a semi-transparent plate are performed. An ablation process is demonstrated to occur on the plate surface in the case of intense radiative heating of the plate by an external source with emission in a limited spectral range. Temperature fields and distributions of heat fluxes in the boundary layer and in the plate are calculated. Calculation results are presented, which allow determining the effect of ablation and reflecting properties of the plate surface on the thermal state of the medium in the system containing the boundary layer and the plate under conditions of plate heating by a high-temperature source of radiation.

Key words: radiation, turbulence, boundary layer, ablation, scattering.

The process of radiative–convective heat transfer on a porous plate with gas injection through the plate was studied in [1–3]. The injected flow was independent of the plate temperature and was defined a priori. In [4], the process of mass supply through the surface into the boundary layer was studied in combination with heat transfer; an ablating plate model was used. Heat transfer in the flow past a semi-transparent plate in the absence of particles in the boundary layer and ablation on the surface was calculated in [5].

In the present study, we consider an adjoint problem of radiative–convective heat transfer in a turbulent flow of an emitting–absorbing and scattering gas–particle medium past a semi-transparent ablating plate. For simplicity, we assume that the vapors of the plate material do not affect the optical and thermophysical properties of the medium, while the presence of particles in the flow does not affect the thermophysical properties and determines the optical properties. The particle size remains unchanged in the course of heat transfer. The optical properties of the medium depend on temperature and radiation wavelength. The specific heat is assumed to be constant, the viscosity and thermal conductivity are linear dependences of temperature, and the density is an inverse dependence of temperature. Heat transfer inside the plate through radiation and heat conduction is taken into account in the direction perpendicular to the plate surface. The optical properties of the plate material depend on the wavelength, and the thermal conductivity is temperature-dependent. The time of heating of the boundary layer is assumed to be much shorter than the time of heating of the plate; hence, heat transfer in the boundary layer can be considered in a quasi-steady approximation. The initial value of the plate temperature is $T_{w0}$; this value is maintained constant in the region $0 < x < x_0$ during the entire heating process. The lower surface of the plate is thermally insulated. The source of radiation, which is a blackbody with a temperature $T_s$, is located outside the boundary layer and emits in a limited spectral range $\Delta$. The medium of the boundary layer is emitting, absorbing, and scattering, while the medium of the plate is emitting and absorbing. The source surface is parallel to the plate surface.

Under the assumptions used, the velocity field in the boundary layer is described by the differential equation

$$\left((1 + \mu_1) f'' + \frac{1}{2} ff'' - f' \frac{\partial f}{\partial \xi} - f'' \frac{\partial f}{\partial \xi}\right)_{\xi = \xi_0} = 0$$

where

- $f$ is the velocity component
- $\mu_1$ is the viscosity
- $\xi$ is the coordinate
- $\xi_0$ is the boundary layer thickness


with the boundary conditions

\[ \eta = 0: \ f = 0, \ \ f' = -f_w, \ \ \eta \to \infty: \ f' \to 1, \]

where \( f \) is the dimensionless stream function, \( f_w = V_w(\text{Re})^{1/2} \), \( V_w = \rho_w v_w/(\rho_{\infty} u_{\infty}) \) is the dimensionless mass flow on the plate surface, the subscripts \( w \) and \( \infty \) correspond to conditions on the plate and in the external flow, \( \eta = \left( \frac{\rho_{\infty} u_{\infty}}{\mu_{\infty} x} \right)^{1/2} \int_0^\rho \frac{\rho}{\rho_{\infty}} \, dy \) and \( \xi = x/L \) are the transverse and longitudinal dimensionless coordinates, \( x \) and \( y \) are the corresponding dimensional coordinates, \( u \) and \( v \) are the streamwise and crossflow components of velocity, respectively, \( \rho \) is the density, \( \mu \) is the viscosity, \( L \) is the length of the calculated region of the plate, \( \text{Re} = \rho_{\infty} u_{\infty} L/\mu_{\infty} \) is the Reynolds number, and the prime means differentiation with respect to the coordinate \( \eta \).

The thermal part of the problem consists of equations and boundary conditions that describe heat transfer in the boundary layer

\[
\frac{\partial}{\partial \eta} \left( \frac{1}{Pr} + \frac{\mu_t}{Pr_t} \frac{\partial \theta}{\partial \eta} \right) + \frac{f}{2} \frac{\partial \theta}{\partial \eta} - \xi f' \frac{\partial \theta}{\partial \xi} = \frac{\text{Sk}}{\text{Re} Pr} \xi \Psi = 0,
\]

\( \xi_0 < \xi < \xi_1, \ 0 < \eta < \infty, \)

\( \xi = \xi_0: \ \theta = \theta_0, \)

\( \eta = 0: \ \theta = \theta_w, \ \eta \to \infty: \ \theta \to 1 \)

and in the plate

\[
\frac{\partial \theta_w}{\partial \text{Fo}} = \frac{\partial}{\partial \xi} \left( \Lambda \frac{\partial \theta_w}{\partial \xi} + \text{Sk}_w \frac{\partial \Phi_w}{\partial \xi} \right), \ \ 0 < \zeta < 1, \ \ \text{Fo} > 0; \]

\( \text{Fo} = 0, \ 0 \leq \zeta \leq 1: \ \theta_w = \theta_{w0}; \)

\( \zeta = 0, \ \text{Fo} > 0: \ \Lambda \frac{\partial \theta_w}{\partial \xi} = \text{Sk}_w (Q - \Phi_w); \)

\( \zeta = 1, \ \text{Fo} > 0: \ \frac{\partial \theta_w}{\partial \xi} = 0. \)

[Note that Eq. (4) is the condition of matching of the heat flux on the interface between the boundary layer and the plate.] In relations (1)–(4), \( \mu_t = \mu_t/\mu \) (\( \mu_t \) is the turbulent viscosity), \( \theta = T/T_{\infty}, \ \theta_w = T_w/T_{\infty} \) (\( T \) and \( T_w \) are the temperatures in the boundary layer and in the plate, respectively), \( \theta_0(\eta) \) is the self-similar solution of the energy equation (2) with radiation being ignored, \( \Phi_w = E_w/(4\sigma T_{w}^4) \), \( E_w \) is the integral (over the spectrum) density of the resultant radiation flux in the plate, \( \zeta = y/H \) (\( H \) is the plate thickness), \( \text{Fo} = a_{\text{t}} H^2/a_{\infty} \) is the Fourier number, \( \text{Pr} = \mu_{\infty}/(\rho_{\infty} a_{\infty}) \) is the Prandtl number, \( \text{Sk} = 4\sigma T_{\infty}^3/\lambda_{\infty} \) and \( \text{Sk}_w = 4\sigma T_{w}^3/\lambda_{\infty} \) are the Stark numbers in the boundary layer and in the plate, respectively, \( \text{Pr}_t \) is the turbulent Prandtl number, \( \Lambda = \lambda_c/\lambda_{\infty} \) (\( \lambda_c \) and \( \lambda_{\infty} \) are the thermal conductivities of the plate material and of the medium in the external flow, respectively), \( a_c \) and \( a_{\infty} \) are the thermal diffusivities of the plate material and of the medium in the external flow, \( \xi_0 = x_0/L \) and \( \xi_1 = x_1/L \) (\( x_0 \) and \( x_1 \) are the boundaries of the calculated region of the plate), and \( \sigma \) is the Stefan–Boltzmann constant.

The dimensionless density of the total heat flux on the plate surface \( Q \) in Eq. (4) is determined by the expression

\[
Q = -\frac{1}{\text{Sk}} \left( \frac{\text{Re}}{\xi} \right)^{1/2} \frac{\partial \theta}{\partial \eta} \bigg|_{\eta=0} + \Phi - \frac{\text{Re Pr}}{\text{Sk}} V_w Q_L,
\]

where \( \Phi = E/(4\sigma T_{\infty}^4), \ E \) is the integral (over the spectrum) density of the resultant radiation flux in the boundary layer, and \( Q_L = q_L/(\rho_{\infty} c_p T_{\infty}) \) (\( q_L \) is the heat of evaporation of the plate material). The expression for dimensionless divergence of radiation flux density in Eq. (2) has the form

\[
\Psi = \int_{\Delta} \frac{T_{\lambda L}(E_{0\lambda} - E_{\Delta \lambda})}{4\sigma T_{\infty}^4} \, d\lambda,
\]

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