Equiparadoxicality of Yablo’s Paradox and the Liar

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Abstract It is proved that Yablo’s paradox and the Liar paradox are equiparadoxical, in the sense that their paradoxicality is based upon exactly the same circularity condition—for any frame $\mathcal{K}$, the following are equivalent: (1) Yablo’s sequence leads to a paradox in $\mathcal{K}$; (2) the Liar sentence leads to a paradox in $\mathcal{K}$; (3) $\mathcal{K}$ contains odd cycles. This result does not conflict with Yablo’s claim that his sequence is non-self-referential. Rather, it gives Yablo’s paradox a new significance: his construction contributes a method by which we can eliminate the self-reference of a paradox without changing its circularity condition.

Keywords Circularity · Equiparadoxical · Liar paradox · T-schema · Yablo’s paradox

1 Introduction

Yablo (1985) introduced a countable sequence of sentences, each of which says that all subsequent sentences in the sequence are untrue—that is, a sequence $Y_1, Y_2, \ldots,$ of sentences such that for every $n \geq 1$, $Y_n$ says that $Y_k$ is untrue whenever $k > n$. Yablo claimed his paradox arises “in the complete absence of self-reference”, and is “not in any way circular” (1993: 251).
Priest argued, on the contrary, that “self-referential circularity is involved in Yablo’s paradox” (1997: 236, Priest’s italic). Priest’s point is based on his observation that the formulation of Yablo’s paradox involves the existence of a fixed-point predicate. His observation caused considerable controversy.¹

We find Priest’s observation unsatisfactory since it obscures the essential features of Yablo’s paradox. The problem is that the existence of the fixed-point predicate is insensitive to whether a sequence is paradoxical or not. As Priest admitted himself, his construction of the fixed-point predicate is related to nothing but the formulation of Yablo’s sequence (Bueno and Colyvan 2003a: footnote 5). To see this, just consider the sequence that is exactly the same as Yablo’s sequence except that each of its sentences says the subsequent sentences are all true. This sequence, according to Priest’s analysis, involves a fixed-point predicate. But it is not paradoxical at all. In that case, how could we say Yablo’s paradox essentially involves the existence of a fixed-point construction?

Different people may understand circularity differently. But if it is supposed to be essential to a paradoxical sequence, it must be proved that the supposed circularity is necessary and sufficient for this sequence to lead to paradox. The kind of circularity inherent in fixed-point structures is clearly not up to this standard. The aim of the present paper is to find a kind of circularity that meets this standard. Surprisingly, the main result we establish in this paper shows that Yablo’s paradox and the Liar paradox are equiparadoxical in the sense that their paradoxicality is based upon exactly the same circularity condition. More specifically, when the sentences are evaluated with respect to points in the (relational) frames, Yablo’s sequence leads to a paradox in the very frames where the Liar sentence does so, and these frames must contain a kind of cycle. More details are given in the next section. Our main result is proved in the third section. In the last section, we point out that our result does not conflict with Yablo’s claim that his paradox is not self-referential (in the narrow sense), but gives Yablo’s paradox a new significance: his construction contributes a method by which we can eliminate the self-reference of a paradox without changing its circularity condition.

2 Basic Notions and the Main Result

Let $\mathcal{L}$ be the first-order language of Peano arithmetic and let $\mathcal{L}^+$ be the language obtained from $\mathcal{L}$ by adjoining a unary predicate $T$. A ground model of $\mathcal{L}^+$ is a pair $\langle \mathfrak{M}, X \rangle$, where $\mathfrak{M} = \langle \mathbb{N},', +, \cdot, 0 \rangle$ is the standard model of $\mathcal{L}$ and $X \subseteq \mathbb{N}$ is the extension of $T$. For a sentence $A$ of $\mathcal{L}^+$, define $X \models A$ ($A$ is true in $\langle \mathfrak{M}, X \rangle$) as usual, so that in particular $X \models T \uparrow A^\uparrow$ iff the Gödel number of $A$ is in $X$.

By an easy diagonalisation, we can find a sentence $L$ in $\mathcal{L}^+$, such that for any $X \subseteq \mathbb{N}$, $X \models L$ iff $X \models \neg T \uparrow L^\uparrow$. So $L$ is equivalent to the sentence that asserts $L$’s untruth. In this sense, $L$ is a formal representation of the Liar sentence (the sentence

¹ For example, Beall (2001) and Ketland (2005) sided with Priest, whereas Bueno and Colyvan (2003a,b, 2011) stood on the opposite side by arguing the fixed point is not necessary to formulation of Yablo’s paradox.