HEAT CONDUCTION AND HEAT EXCHANGE IN TECHNOLOGICAL PROCESSES

ENGINEERING METHODS OF CALCULATION OF DIFFERENT REGIMES OF HEATING OF THERMALLY MASSIVE OBJECTS IN METALLURGICAL HEAT TECHNOLOGIES UNDER COUNTERCURRENT CONDITIONS. 2. RADIATIVE AND RADIATIVE-CONVECTIVE COUNTERCURRENT HEATING

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An approximate procedure of mathematical modeling of radiative and radiative-convective countercurrent heat exchange in metallurgical units has been given.

Heat exchange of solid bodies and a gas in their countermotion is quite frequently realized in various branches of technology and primarily in metallurgy. Examples of such processes are heating of round billets in annular furnaces before piercing, heating of a burden in blast furnaces, cooling of pellets in stack-type coolers, etc. In exact solution, these problems are considered only in a linear formulation, whereas the processes themselves are nonlinear. This is because of the difficulties appearing in their nonlinear mathematical modeling. The arising complications can be successfully solved with the method of equivalent sources [1–3], which has been adequately tested in problems of concurrent and countercurrent heat exchange (CCHE).

Radiative Countercurrent Heat Exchange. Let us consider the problem of countercurrent symmetric radiative heating of thermally massive bodies of a base shape in the following formulation [1–3]:

\[
\frac{1}{\rho_m} \frac{\partial}{\partial \rho} \left( \rho_m \frac{\partial \theta}{\partial \rho} \right) = \frac{\partial \theta}{\partial \tau},
\]

\[
\left. \frac{\partial \theta}{\partial \rho} \right|_{\rho=1} = Sk \left[ \theta_g^4 (\tau) - \theta_s^4 (\tau) \right], \quad \left. \frac{\partial \theta}{\partial \rho} \right|_{\rho=0} = 0,
\]

\[
\frac{d \theta_g}{d \tau} = Sk \left[ \theta_g^4 (\tau) - \theta_s^4 (\tau) \right] n_m,
\]

\[
\theta (\rho, 0) = \theta_0 = \theta' = \text{const}, \quad \theta_g (0) = \theta_g' = 1,
\]

where we have

\[
\theta_s (\tau) = \theta (1, \tau); \quad \theta (\rho, \tau) = \frac{T_m (\rho, \tau) - T_m'}{T_g' - T_m}; \quad n_m = (1 + m) n; \quad \rho = \frac{r}{R};
\]

\[
\tau = \frac{at}{R^2}; \quad B_i = \frac{\alpha R}{\lambda}; \quad \theta (\rho, \tau) = \frac{T (\rho, \tau)}{T_g}; \quad \theta_g (\tau) = \frac{T_g (\tau)}{T_g''}; \quad Sk = \frac{\sigma_s T_g^3}{\lambda}.
\]  

(5)

The parameter \(n_m\) allows for the relation of the heat capacities of a solid body and a gas moving in opposition.

In the method of equivalent sources, which represents a combination of the method of successive approximations and the integral methods, it is taken that the inertial step of heating is completed after a certain time \(\tau_0\) and an ordered step of warmup over the entire body begins.

In the first inertial step of heating \((0 \leq \tau \leq \tau_0\) and \(\beta(\tau) \leq \rho \leq 1)\), we use the ready solution of problem (1), (2) by the method of equivalent sources [1–3]:

\[
\theta_1 (\rho, \tau) = \theta' + \left[ \theta_{1s} (\tau) - \theta' \right] \left[ \frac{\rho - \beta (\tau)}{1 - \beta (\tau)} \right]^2.
\]

(6)

\[
l (\tau) = 1 - \beta (\tau) = \frac{2}{Sk} \frac{\theta_{1s} (\tau) - \theta'}{\theta_{1g}^4 (\tau) - \theta_{1s}^4 (\tau)}.
\]

(7)

The surface temperature \(\theta_{1s}(\tau)\) or the temperature difference \(\Delta \theta_1 (\tau) = \theta_{1s}(\tau) - \theta'\) is determined by solution of the differential equation

\[
\frac{d}{d\tau} \left[ \Delta \theta_1 (\tau) l (\tau) \right] = \frac{6 (1 + m) \Delta \theta_1 (\tau)}{l (\tau)}.
\]

(8)

Equation (8) must be considered simultaneously with expression (7) and thermal-balance condition (3) but, taking into account the usual rapidity of the inertial period for the majority of metallurgical objects, we can simplify determination of the functions \(\theta_{1s}(\tau)\) and \(\theta_{1g}(\tau)\), assuming that the advance of the warmup front \(l(\tau)\) satisfies a certain existing law, which is represented in this case by the formula [3]

\[
l (\tau) = \sqrt{6 (1 + n)} \tau, \quad \tau_0 = [6 (1 + n)]^{-1}.
\]

(9)

Therefore, the solution of Eq. (8) has the form

\[
\Delta \theta_1 (\tau) = \sqrt{\frac{\tau}{6 (1 + m)}}, \quad \theta_{1s} (\tau) = \theta' + \sqrt{\frac{\tau}{6 (1 + m)}}.
\]

(10)

From relation (7) we find

\[
Sk \left[ \theta_{1g}^4 (\tau) - \theta_{1s}^4 (\tau) \right] = \frac{2 \Delta \theta_1 (\tau)}{l (\tau)}.
\]

(11)

Then conditions (3) and (4) with account for (9)–(11) lead to the following expression for the gas temperature:

\[
\theta_{1g} (\tau) = 1 + \frac{n}{3} \Delta \theta_1 (\tau) l (\tau) = 1 + \frac{n \tau}{3}.
\]

(12)

In the second (ordered) step \((\tau_0 \leq \tau < \tau_\ast\) and \(0 \leq \rho \leq 1)\), the resolving equation of the method of equivalent sources is taken in the form of [1–3]. Integrating this equation [1–3] with respect to \(\rho\) and using boundary condition (2), we arrive at the solution

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