RADIATIVE HEAT EXCHANGE IN COMBUSTION PROCESSES

NUMERICAL SOLUTION OF THE RADIATIVE-TRANSFER EQUATION FOR AN ABSORBING, EMITTING, AND SCATTERING MEDIUM WITH A COMPLEX 3-D GEOMETRY

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A method is proposed for numerically solving the integro-differential, radiative-transfer equation with the use of its piecewise-analytic solutions obtained by the discrete-ordinate method and grids constructed by the finite-element method. The advantages of the method proposed and some results of calculation of the radiative-transfer characteristics for one-, two-, and three-dimensional problems are discussed.

Radiative energy transfer is of crucial importance in many natural and technical processes of energy exchange. This pertains equally to high-temperature processes (combustion of organic fuels, thermal treatment of metals, high-temperature synthesis and pyrolysis in chemical technologies, etc.) where the radiative energy transfer accounts for 90% or more of the total energy exchange (see, for example, [1–4]) and to the processes occurring at lower atmospheric temperatures [5, 6]. It is known that an exact estimation of the characteristics of heat and mass transfer in technological processes allows one to obtain a significant economical effect, i.e., to increase the quality and functional characteristics of products and decrease their cost, as well as to make for good environmental conditions and to conserve material, energy, and manpower resources. For example, the temperature fields of many technological processes (several tens of them are described in [1, 2]) occurring at temperatures from 125 to 1600°C should be calculated with an accuracy of ±1–2°C.

Radiative heat exchange plays a dominant role in the total heat exchange in high-temperature processes in gaseous media. The accuracy of estimation of the temperature fields of such media is primarily dependent on the correctness of calculation of the radiative-transfer characteristics. This is also very important for optimization of the heating of steel products having a different geometry in ring furnaces with a moving bottom in which the working temperatures can reach 1200°C.

Mathematical Model. It is difficult to calculate the characteristics of radiative heat transfer in selectively emitting, absorbing, and scattering media because, in this case, it is necessary to take into account the multiple processes of reradiation on solid particles, the selectivity of the radiation of gas components, and the temperature inhomogeneity and complex configuration of the radiating volume. The correctness of estimation of the radiative-heat-exchange characteristics depends, to a large extent, on the correctness of solution of the radiative-transfer equation [7–9]. In the case of a local thermodynamic equilibrium, this equation defines the law of conservation of radiant energy in the process of its propagation in an absorbing, emitting, and scattering medium:

\[ I(\mathbf{r}, \mathbf{l}) + [\chi_\lambda(\mathbf{r}) + \sigma_\lambda(\mathbf{r})] I_\lambda(\mathbf{r}, \mathbf{l}) = \chi_\lambda(\mathbf{r}) B_\lambda(T(\mathbf{r})) + \frac{\sigma_\lambda(\mathbf{r})}{4\pi} \int_{4\pi} p_\lambda(\mathbf{r}, \mathbf{l}, \mathbf{l}') I_\lambda(\mathbf{r}, \mathbf{l}') d\Omega'. \]  

(1)

The boundary conditions for Eq. (1) are determined by the radiation and reflection processes occurring on the boundary surfaces of the medium and can be written, in the general case, as [9]

\[
I_{\lambda}(P, l)|_{(l \cdot n)<0} = I_{0\lambda}(P, l) + \frac{1}{2\pi} \int \rho_{\lambda}(P, l', l) I_{\lambda}(P, l') \, d\Omega'.
\]

Having determined the radiation-intensity field from Eqs. (1) and (2), we may determine two more energy quantities necessary for calculating the temperature of the medium — the volume density of the radiation sources/heat flows at each point of the medium

\[
d\mathbf{Q}_{\tau} = \int_{0}^{\infty} \chi_{\lambda}(r) \left( 4\pi B_{\lambda}(T(r)) - \int I_{\lambda}(r, l) \, d\Omega \right) d\lambda.
\]

and the local densities of the resulting radiant flux on the heat-absorbing surfaces (if they exist)

\[
q_{w}^I(P) = \int_{0}^{\infty} \left( \int I_{\lambda}(P, l) \, (l \cdot n) \, d\Omega - \pi B_{\lambda}(T_{w}(P)) \right) d\lambda.
\]

Examples of such surfaces include the lining of a furnace and the surfaces of steel products heated in this furnace.

**Brief Review of Modern Methods of Solving the Radiative-Transfer Equation.** At present there are a fairly large number of different methods of solving Eq. (1) with boundary conditions (2): the Monte Carlo method [10]; the approximations of spherical harmonics [11], radiation elements [12], and characteristics [9, 13]; zonal methods [8], and others. A recent trend in the methodology of solving the radiative-transfer equation is the combination of the discrete-ordinate method [7] with the finite-difference method [14, 15] or the finite-element method [3, 16]. The popularity of this approach to the solution of the radiative-transfer equation is explained by the fact that the computational algorithm used in it is relatively simple and compatible with the computational schemes used in the case of different mechanisms of radiative transfer. There are also a number of other methods of attack of this problem; however, a sufficiently reliable and efficient method of solving the radiative-transfer equation is absent at the moment. Each of the existing methods has disadvantages that limit the range of its application. For example, the method of finite elements or finite volumes can be used for solving a limited range of ordinary differential equations (of the first order, the hyperbolic type) for nonuniform high-temperature heat flows; otherwise, physically incorrect results could be obtained (e.g., negative values of the radiation intensity).

To demonstrate problems that could arise in the process of numerical solution of the radiative-transfer equation, we write the radiative-transfer equation in the one-dimensional formulation with account for the isotropic scattering. In this case, Eq. (1) is conveniently written in the form

\[
\mu \frac{dI}{dx} + \alpha I(x, \mu) = Y.
\]

Let us solve Eq. (5) in the finite-difference approximation by the explicit and implicit finite-difference schemes for the segment \([1 \rightarrow 2]\) of length \(\Delta l\) (the arrow designates the radiation-propagation direction) with homogeneous optico-physical properties:

\[
I_{2}^{an} = I_{1} \exp \left[ -\tau_{at} \right] + \left( 1 - \exp \left[ -\tau_{at} \right] \right) \frac{Y}{\alpha},
\]

\[
I_{2}^{im} = I_{1} \frac{2 - \tau_{at}}{2 + \tau_{at}} + \frac{2Y\Delta l}{\mu (2 + \tau_{at})},
\]

\[
I_{2}^{ex} = I_{1} \frac{1}{1 + \tau_{at}} + \frac{Y\Delta l}{\mu (1 + \tau_{at})}.
\]