A direct relationship between the theory of inverse problems of mathematical physics and the theory of structural properties of dynamic systems is established based on which the inverse problems of mathematical physics and heat conduction are classified and some of the works on them are reviewed.

Introduction. The foundations of the theory of inverse problems of mathematical physics were laid in the 1950s–1960s in the works of A. N. Tikhonov, M. M. Lavrent’ev, I. M. Gel’fand, B. M. Levitan, M. G. Krein, V. A. Marchenko, L. D. Faddeev, and many other mathematicians. The special properties of inverse problems are that, unlike primal problems, they do not possess the property of correctness in the sense of Adamar. In this connection, A. N. Tikhonov and his followers have developed the theory of regularization of ill-posed problems and have proposed stable methods of their solution [1–12].

In thermophysics, inverse problems occur as problems of either diagnostics of the thermophysical parameters and internal and (or) boundary sources of the processes of transfer or control and synthesis of the above parameters and sources. We emphasize that the "investigation methodology based on solution of inverse problems is one new line in studying heat- and mass-exchange processes and in processing and optimizing thermal regimes of technical objects and technological processes [13]." The problems and methods of solution of the inverse problems of heat exchange have been presented in detail in [14] (this monograph is now classical).

In the present work, we review the basic classes of inverse problems of mathematical physics and, in particular, inverse problems of heat conduction; the inverse problems are organized in accordance with the classification (proposed in [15]) of inverse problems of mathematical physics. The basis for the classification used is the scheme of cause-and-effect relations of dynamic systems. The notion of a dynamic system is fundamental for primal problems of mathematical physics. The theory of structural properties and characteristics of systems, such as controllability, observability, reversibility, realizability, and others, has also been developed within the framework of dynamic systems [16–29]. It turned out that these characteristics are directly related to the formulation of a number of classical inverse problems of mathematical physics and inverse problems of heat conduction [15]. Thus, the classification of inverse problems of mathematical physics that is presented in this review links the theory of inverse problems to the theory of dynamic systems in the space of states, which contributes to the interdisciplinary exchange of results and, in particular, to the use of the methods of the theory of dynamic systems in the theory of inverse heat-conduction problems.

It should be noted that investigations of the inverse problems of mathematical physics are the focus of numerous works, including monographs (see, e.g., [1, 2, 8, 13, 14, 30–56]). Therefore, in this review, we have restricted our consideration to only the part of the works that were not mentioned in the above monographs but are of interest from the viewpoint of the theory of systems and inverse problems of mathematical physics. In this work, we do not consider inverse problems in the stationary formulation and a wide class of problems associated with the shaping of bodies.
**Basic Notions of System Theory.** In describing inverse problems in general form, it is expedient to use the set-theoretical apparatus of a mathematical abstract system theory [16–18, 57]. The result of the abstract system theory, best suited to our purposes, is rigorous formalization of the notion of cause-and-effect relations of systems considered within the framework of some mathematical models or others. Therefore, the basis for the classification of inverse problems used here is the scheme of cause-and-effect relations of an abstract dynamic system. Since most of the inverse problems of mathematical physics are currently described in terms of distributed dynamic systems [2, 14, 30–35], the classification developed further may serve as a basis for the hierarchic structure with a more detailed taxonomy of inverse problems and inverse heat-conduction problems, for example, in thermophysical and other signs [31–33, 35, 41, 48, 58–60].

We briefly describe set-theoretical structures of the abstract system theory that will be necessary in the future. This will enable us to emphasize the fundamental character and universality of such notions as "dynamic system," "input," "state," "output," and "reaction of the system (input–output map)." In turn, the basic system properties — controllability, observability, reversibility, realizability, and structural and parametric identifiability — are adequately described precisely in terms of the above notions and are of crucial importance in formulating and classifying the inverse problems of mathematical physics. The algebraic methods of the abstract system theory, thus far developed only for linear systems in detail, are also of great interest [16, 19–22, 61]. We note that assimilation of algebraic system-structural methods by the heat-transfer theory has been reflected in [19, 62].

In the general case [17], the abstract system $S$ is determined as a subset of the Cartesian product $\Omega \times \Gamma$ of certain sets $\Omega$ and $\Gamma$:

$$S \subseteq \Omega \times \Gamma.$$  \hfill (1)

The components $\Omega$ and $\Gamma$ of the Cartesian product $\Omega \times \Gamma$ are respectively called the input and output objects of the system $S$. For time-variable systems the input and output objects are sets of functions of the time $t$, $t \in \Theta$, i.e., $\Omega = U^\Theta$ and $\Gamma = Y^\Theta$, where $U$ is the set of values of the input quantities, $Y$ is the set of values of the output quantities, and $\Theta$ is the linearly ordered set of the instants of time. The set $\Theta$ can be discrete or continuous; also, we do not rule out a combined variant. The functions of $\Omega$ are called the inputs of the system $S$, whereas the functions of $\Gamma$ are the outputs.

Relation (1) may be interpreted as a generally multivalued map $S : \Omega \rightarrow \Gamma$ setting up a correspondence between the causes (inputs) and effects (outputs). The multivaluedness of $S$ is related to certain internal parameters of the system. In the abstract system theory, these parameters are called the states of a system [16, 17]. For a prescribed system $S$ we can always construct the object of global states — such a set $X$ that the state $x \in X$ ensuring the equality $\gamma = R(x, \omega)$ exists for any input–output pair $(\omega, \gamma)$, $(\omega, \gamma) \in S$. The map $R : X \times \Omega \rightarrow \Gamma$ is the global reaction of the system $S$ [17].

The notion of a time-causal system holds one central position in the abstract system theory. Next, following [16], we identify the notion of a causal system and a dynamic system. The future behavior of a dynamic system exerts no influence on its past, and this imposes certain restrictions on the structure of the map of $R$. Such restrictions are usually described [16, 17] in terms of the transient function, the output map, and the contraction $R_{[\tau,t]}$ of the global reaction to the time interval $[\tau, t]$, where $\tau$ is the initial instant of time and $t$ is the running instant.

Let us denote the contraction of the input $u(\cdot)$ to the time interval $[\tau, t]$ by $u_{[\tau,t]}$. Then the value of the transient function $\varphi : \Theta \times \Theta \times X \times \Omega_{[\tau,t]} \rightarrow X$ coincides with the state

$$x(t) = \varphi(t, \tau; x(\tau), u_{[\tau,t]})$$  \hfill (2)

of the dynamic system at the instant of time $t$ if the dynamic system was in the state $x(\tau)$ at the initial instant $\tau$, $\tau < t$, and the input $u_{[\tau,t]}$ acted on it. The transient function possesses a number of characteristic properties [16] that are not the focus of our attention here.

The output map $\eta : \Theta \times X \times U \rightarrow Y$ determines the value of the output at the running instant of time $t$:

$$y(t) = \eta(t, x(t), u(t)),$$  \hfill (3)