HEAT CONDUCTION AND HEAT TRANSFER IN TECHNOLOGICAL PROCESSES

MATHEMATICAL MODEL OF A THERMOSTATING COATING WITH A THERMOELECTRIC MODULE

V. S. Zarubin, G. N. Kuvyrkin, and I. Yu. Savel’eva

UDC 536.2; 621.362

On the basis of a variational formulation of the problem of stationary heat conduction in a heterogeneous solid, a mathematical model of a fragment of a flat heat-insulating layer containing a thermoelectric module has been constructed. This model has been used to establish conditions under which, when fulfilled, the heat-insulating layer can serve as a thermostating coating for an object with a given fixed temperature under convective-radiative heat exchange on the outer surface of the fragment under consideration. The results of the qualitative analysis of the proposed model are presented.

Keywords: heat insulation, thermostating Peltier effect, thermoelectric module.

Introduction. It is known that the resistance of a homogeneous flat heat-insulating layer of thickness \( h \) with heat conductivity coefficient \( \lambda \) is equal to the ratio \( h/\lambda \). Including, in the heat-insulating layer, elements exhibiting the thermoelectric Peltier effect when an electric current passes through them permits influencing the effective value of the thermostatic resistance of such a layer by changing the current intensity and direction and thus controlling the heat transfer process in this layer [3, 4]. The Peltier effect consists of releasing or absorbing (depending on the electric current direction) the heat power at the place of contact of two heterogeneous materials

\[
Q_e = 2e \cdot TI.
\]  

The value of the thermoelectric coefficient of the contacting pair of materials \( e \) can be fairly high when using semiconductor elements contacting metal conductors and forming thermoelectric modules [2, 5–7]. This fact widens the possibilities of using thermoelectric modules in various fields of engineering [8–10].

One of the aims of controlling the thermal resistance of the heat-insulating layer is to increase it without bound. When this aim is achieved, the heat-insulating layer with thermoelectric modules serves as a thermostating coating completely insulating the covered object with a fixed temperature against the thermal action of the environment. Establishing the relation between the diagnostic variables to provide thermostating conditions under convective-radiative heat exchange on the outer surface of such a heat-insulating layer is a topical problem as applied to various fields of engineering.

Formulation of the Problem. We consider a fragment of a heat-insulating layer of thickness \( h \) having in plan an area \( F \) and containing one thermoelectric module (Fig. 1). This module represents a semiconductor element 1 connected to two thin commutating metal plates 2 and 3 with commutating conductors 2" and 3". When electric current flows in the chosen positive direction indicated in Fig. 1 by an arrow, thermal energy is released and absorbed, respectively, at the places of contact with conductors 2" and 3". When the current direction is changed, the thermal energy is released and absorbed, respectively, at the places of contact with conductors 3" and 2".

The commutating plate 2 is separated from the environment with temperature \( T_\infty \) by an electroinsulating layer 2' of thickness \( h_2 \) with a heat conductivity coefficient \( \lambda_2 \). The intensity of convective heat exchange with the environment determines the heat transfer coefficient \( \alpha \). The outer surface of this electroinsulating layer with temperature \( T_e \) has an emissivity \( \varepsilon \) and an absorption coefficient \( A \) with respect to the radiation flux of density \( q \) incident from the outside. The electroinsulating layer 3' of thickness \( h_3 \) with a heat conductivity coefficient \( \lambda_3 \) separates plate 3 from the thermostated object with a given temperature \( T_0 \).

The side surfaces of the chosen fragment are assumed to be ideally heat-insulated.

N. É. Baumann Moscow State Technical University, 5 2nd Baumanskaya Str., Moscow, 105005, Russia; email: fn2@bmstu.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 88, No. 6, pp. 1328–1335, November–December, 2015. Original article submitted December 17, 2014.
Let the commutating plates 2 and 3 and conductors 2" and 3" be made from an adequate heat-conducting material. This permits considering these plates and conductors to be homogeneous in their volume, as well as the temperature \( T_2 \) of plate 2 to be equal to the temperature of conductor 2", and the temperature \( T_3 \) of plate 3 to be equal to the temperature of conductor 3". The cross-sectional areas of the semiconductor element 1 and conductors 2" and 3" are assumed to be equal. It is assumed that the thermoelectric coefficient has a constant value \( (e_\ast = \text{const}) \), which permits neglecting the Thompson effect \([1, 2]\), and that the thermoelectromotive force arising in the electric circuit from heterogeneous conductors at a nonuniform temperature distribution and characterizing the Seebeck effect \([1]\) is comparatively small and its value can be neglected.

The electrical resistances of plates 2 and 3 \( R_2 \) and \( R_3 \) are determined with account for the resistances of the commutating conductors 2" and 3" and the resistances of the places of their contact with the semiconductor element 1.

**Mathematical Model of the Fragment of the Heat-Insulating Layer.** The mathematical model of the temperature state of the fragment of the heat-insulating layer is constructed on the assumption that the temperature distribution in this layer is steady with the use of the above assumptions. To construct this model, a variational formulation of the linear problem of the stationary heat conductivity in a heterogeneous body is used \([11, 12]\). This problem contains the functional

\[
J[T] = \frac{\lambda_1}{2} \int_V \left( \nabla T(M) \right)^2 dV(M) + \frac{\lambda_2}{2} \int_{V_1} \left( \nabla T_1(N) \right)^2 dV(N)
+ \frac{\lambda_3}{2} \int_{V_2} \left( \nabla T_2(P) \right)^2 dV(P) - \rho_1 \left( I/F_1 \right)^2 \int_{V_1} T_1(N) dV(N) + \alpha F \left( T_2/2 - T^\ast \right) T_2
+ \varepsilon \sigma_0 T_2^4 F / 8 + \left( T_3/2 - T_0 \right) \lambda_3 T_2 F / h_3 - T^2 \left( R_2 T_2 + R_3 T_3 \right) - e_\ast \left( T_2^2 - T_3^2 \right),
\]

where \( V = (F - F_1)h, \; V_1 = F_1 h_1, \) and \( V_2 = F h_2 \) are the volumes of the heat insulator, the semiconductor element 1, and the electroinsulating layer 2' (Fig. 1), \( T(M) (M \in V), T_1(N) (N \in V_1) \) and \( T_2(P) (P \in V_2) \) are the temperature distributions in volumes \( V, V_1 \), and \( V_2 \), \( T^\ast = T_0 \pm Aq/\alpha \), and \( \sigma_0 = 5.67 \times 10^{-8} \; \text{W} / (\text{m} \cdot \text{K}) \) is the Stefan–Boltzmann constant. The sign before the last term on the right side of formula (2) written with account for equality (1) with the chosen positive direction of electric current of intensity \( I \) (Fig. 1) corresponds to the heat release at the place of contact with temperature \( T_2 \) and its absorption at the place of contact with temperature \( T_3 \).

It is allowed to use functional (2) in the cases of continuous and piecewise continuous temperature distributions in regions corresponding to volumes \( V \) and \( V_1 \). Let us assume, as allowed temperature distributions, the distributions linear in thickness \( h \) of the heat-insulating layer and in thickness \( h_2 \) of the electroinsulating layer 2'

\[
T(z) = T_3 + (T_2 - T_3) z/h \quad \text{and} \quad T'_2 = T_2 + (T_3 - T_2) z_2/h_2,
\]

where \( z \) and \( z_2 \) are coordinates reckoned from the direction perpendicular to the surface of the considered fragment of the heat insulator from plates 3 and 2, respectively, and \( T_3 \) is the temperature of the outer surface of the electroinsulating layer 2'.

---

**Fig. 1.** Design diagram of the fragment of the heat-insulating layer: 1) thermoelectric element; 2 and 3, external and internal commutating plates; 2' and 3', outer and inner electroinsulating layers; 2" and 3", commutating conductors. The heat insulator is marked by fine cross-hatching.