MHD MIXED CONVECTION FLOW IN A ROTATING CHANNEL IN THE PRESENCE OF AN INCLINED MAGNETIC FIELD WITH THE HALL EFFECT

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A numerical study of an oscillatory unsteady MHD flow and heat and mass transfer in a vertical rotating channel with an inclined uniform magnetic field and the Hall effect is carried out. The conservation equations of momentum, energy, and species are formulated in a rotating frame of reference with inclusion of the buoyancy effects and Lorentz forces. The Lorentz forces are determined by using the generalized Ohm law with the Hall parameter taken into account. The obtained coupled partial differential equations are nondimensionalized and solved numerically by using the explicit finite difference method. The effects of various model parameters, like the Hall parameter, Hartmann number, wall suction/injection parameter, rotation parameter, angle of magnetic field inclination, Prandtl number, Schmidt number, etc., on the channel velocities, skin friction coefficients, Nusselt number, and the Sherwood number are examined. It is found that the influence of the Hartmann number and Hall parameter on the channel velocities and skin friction coefficients is dependent on the value of the wall suction/injection parameter.

Keywords: MHD, Hall effect, inclined magnetic field, rotating channel, heat and mass transfer.

Introduction. The study of magnetohydrodynamic (MHD) flows continues to attract the attention of applied mathematics and engineering sciences communities owing to their considerable practical applications, such as cooling systems, MHD generators, MHD accelerators, and medical instruments. In aerospace sciences, the study of MHD flows is of importance for improving the aerodynamic performance of reentry vehicles and designing MHD thrusters. The material processing industry also employs MHD devices for better quality of the final products. As the MHD heat and mass transfer processes are of considerable interest, several corresponding analytical investigations have been carried out [1–6], where the influence of the viscous, buoyancy, and magnetic forces on the velocity and temperature profiles were taken into account. Chaudhary and Sharma [7] considered the combined heat and mass transfer in a laminar mixed convection flow from a vertical surface with an induced magnetic field. Hydromagnetic unsteady mixed convection and mass transfer past a vertical porous plate were investigated by Sharma and Chaudhary [8]. An MHD unsteady flow and heat transfer over a flat plate with the Navier slip and Newtonian heating were studied by Makinde [9].

In all of the above investigations, a magnetic field was applied perpendicularly to the surface under study. However, in practical situations, it may not always be possible to have such a magnetic field. In numerous applications, including MHD power generation, astrophysics, and material processing, a magnetic field may act obliquely to the flow. Seth and Ghosh [10] initiated the study of an oscillatory Hartmann flow in a rotating channel in the presence of an inclined magnetic field. An unsteady hydromagnetic flow in a rotating system in the presence of an inclined magnetic field was studied by Ghosh [11, 12]. Guria et al. [13] considered an oscillatory MHD Couette flow in a rotating two-plate channel in the presence of an inclined magnetic field with the fixed upper plate and nontorsionally oscillating lower plate. A steady MHD Couette flow in a rotating system in the presence of an inclined magnetic field with an induced magnetic field taken into account was investigated by Seth et al. [14].

In all of these studies, the effect of the Hall currents was not taken into consideration. In the presence of strong magnetic fields, the Hall effect becomes an important mechanism for electrical conduction in ionized gases and plasmas. Unlike metals, the number density of charge carriers in ionized gases is low, which results in anisotropic behavior of the
electrical properties. Hence, a current is induced in the direction normal to both the electric and magnetic fields. The Hall effect has important engineering applications, such as the Hall generators, Hall probes, and Hall effect thrusters used for space missions. Sato [15] and Sherman and Sutton [16] were the first to investigate the hydromagnetic flow of an ionized gas between two parallel plates with the Hall effect taken into account. The effect of the Hall current on an unsteady MHD free convection flow along a vertical flat plate was studied by Katagiri [17], Hosseini and Mohammad [18], and Pop and Watanabe [19]. An unsteady free convection flow over a vertical plate due to the combined effects of thermal and mass diffusion and the Hall currents was considered by Aboeldahab and Elbarbary [20] and Takhar, Roy, and Nath [21]. A hydromagnetic unsteady mixed convection flow and mass transfer past a vertical porous plate immersed in a porous medium with and without a heat source was investigated by Sharma, Jha, and Chaudhary [22].

The governing flow equations can be written in a rotating frame of reference by consideration of the Coriolis force. A lot of researchers have focused their studies on the analysis of a rotating flow with the Coriolis effect taken into account. Ghosh and Pop [23], Hayat, Nadeem, and Asghar [24], and Seth, Nandkeolyar, and Ansari [25] studied the effects of the Hall currents on an MHD Couette flow of a viscous, incompressible, electrically conducting fluid in a rotating system for different kinds of fluids. Recently, an unsteady hydromagnetic Couette flow of such a fluid in a rotating system in the presence of an inclined magnetic field with the Hall current taken into account was studied by Seth, Nandkeolyar, and Ansari [26]. They also studied a hydromagnetic Couette flow in a rotating system with porous plates, including the Coriolis effect [27]. Mandal, Mandal, and Chaudhury [28] and Ghosh, Beg, and Narahari [29] focused their research on an MHD Couette flow in a rotating environment, considering the combined effects of the Coriolis force, Hall currents, and convective heat transfer.

Thus, the main objective of the present investigation is to study the effect of the Hall currents with the buoyancy forces on an oscillatory flow of a viscous, incompressible, electrically conducting fluid in a rotating porous channel in the presence of a uniform magnetic field applied at an angle with the positive direction of the axis of rotation. The governing equations are solved numerically by the explicit finite difference schemes, and the effects of various pertinent parameters on the flow heat and mass transfer characteristics are investigated and discussed.

**Mathematical Formulation.** We consider an unsteady, incompressible, viscous flow of an electrically conducting fluid confined between two infinite nonconducting vertical parallel plates. The distance between the porous plates of the vertical channel is denoted \( d \). A Cartesian coordinate system is introduced, so that the \( x \) axis is directed vertically upwards in opposition to the direction of the gravity force and the \( z \) axis is perpendicular to the parallel plates. The schematic configuration of the problem is shown in Fig. 1. The plates are subjected to constant injection and suction with velocity \( w_0 \) at the left and right plates, respectively. The temperature at the left plate is considered oscillatory with a frequency \( \omega' \) whereas the oscillations in the \( x \) component of the velocity take place at the right plate. A strong uniform magnetic field inclined at an angle \( \theta \) with respect to the \( x \) axis is applied to the channel, which is rotating with an angular speed \( \Omega' \) about the \( z \) axis.

The strong magnetic field causes the generation of Hall currents, which in turn give rise to the magnetic forces acting on the moving fluid. The Hall effect is taken into account by introducing the Hall parameter \( \sigma = \omega_x r_x \). In the absence of any external electric field, the generalized Ohm law is expressed as \( \mathbf{J} = \frac{m}{B_0} (\mathbf{J} \times \mathbf{B}) = \sigma (\mathbf{V} \times \mathbf{B}) \), where \( \mathbf{J} = (J_x, J_y, 0) \), \( \mathbf{B} = (B_0 \sin \theta, 0, B_0 \cos \theta) \), and \( \mathbf{V} = (u, v, w) \). Using the generalized Ohm law, we obtain

\[
J_x = \frac{\sigma B_0 \cos \theta}{1 + m^2 \cos^2 \theta} (v + mu \cos \theta - mw \sin \theta), \quad J_y = \frac{\sigma B_0}{1 + m^2 \cos^2 \theta} (mv \cos^2 \theta - u \cos \theta + w \sin \theta).
\]

Now with the Lorentz forces \((\mathbf{J} \times \mathbf{B})/\rho\), viscous forces, channel rotation effect (Coriolis force), and the buoyancy effects taken into account, the governing equations for continuity, momentum, energy, and species can be expressed as

\[
\frac{\partial w}{\partial z} = 0 \rightarrow w = w_0, \quad (1)
\]

\[
\frac{\partial u}{\partial t} + w_0 \frac{\partial u}{\partial z} = \nu \frac{\partial^2 u}{\partial z^2} + 2\Omega' v + \frac{\sigma B_0^2 \cos \theta}{\rho(1 + m^2 \cos^2 \theta)} (mv \cos^2 \theta - u \cos \theta + w_0 \sin \theta)
+ g \beta (T' - T_R) + g \beta' (C' - C_R), \quad (2)
\]