A fast memoryless interval-based algorithm for global optimization

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Abstract  We present a global optimization algorithm of the interval type that does not require a lot of memory and treats standard constraints. The algorithm is shown to be able to find one globally optimal solution under certain conditions. It has been tested with many examples with various degrees of complexity and a large variety of dimensions ranging from 1 to 2,000 merely in a basic personal computer. The extensive numerical experiments have indicated that the algorithm would have a good chance to successfully find a good approximation of a globally optimal solution. More importantly, it finds such a solution much more quickly and using much less memory space than a conventional interval method. The new algorithm is also compared with several noninterval global optimization methods in our numerical experiments, again showing its clear superiority in most cases.

Keywords  Global optimization · Interval-based algorithm · Memoryless · Constraints

1 Introduction

Many important real world problems aim at finding the globally optimal value of an objective function \( f(x) \) and at least one global optimizer over a bounded multidimensional interval domain \( X \) in \( \mathbb{R}^n \), possibly subject to some equality and inequality constraints. Mathematically the problem is stated as

\[
\begin{align*}
\text{minimize} \quad & f(x), \\
\text{subject to} \quad & h(x) = 0, \quad g(x) \leq 0, \quad x \in X.
\end{align*}
\]  

(1)

The global problem presents a number of more difficult challenges than local optimization problems. The most difficult issue is perhaps the lack of a single verifiable sufficient condition for a globally optimal solution unless it is a very special case. Thus, either a global
behavior of \( f(x) \) (e.g., Lipschitz constant [5]) is used or the entire search domain is examined by global search algorithms.

Stochastic algorithms search the whole domain only in a probabilistic fashion so that at most they can yield a good estimate of a globally optimal solution in a probabilistic sense. Thus, when such a particular search program stops after a finite number of steps, there is no reliable way to judge the quality of the estimated solution. They are often considered as heuristic. However, stochastic search methods (such as the simulated annealing methods and genetic algorithms) have been more popular choices than interval methods because of their simplicity of implementation, relative quickness for reaching an approximate solution, less memory demands, and a wider range of applicable problems. Many stochastic search methods have been designed for solving unconstrained problems. Under the presence of constraints, their performance deteriorates further and there are even fewer theoretical justifications.

Deterministic algorithms offer attractive alternatives for solving problem (1). They are generally based on the idea of branch and bound [15]. Among them, interval methods offer both sound theoretical foundation and reliable numerical solutions [20]. Under the framework of interval branch and bound, a number of advantages are well known. (1) It guarantees convergence to all global solutions under fairly weak assumptions. (2) It offers reliable stopping criteria so that the algorithm does not have to run longer than necessary. (3) It is numerically robust and handles round-off errors conveniently and effectively. (4) It handles constraints with relative ease and without jeopardizing theoretical justifications. Despite such attractive features of the interval method, most published reports on their applications seem to be generally limited to optimization problems in low dimensions (say, much less than 100 according to our recent survey of literature). Obviously, there are three major concerns in solving large dimensional problems: large amount of memory space, slow speed of convergence, and requirement of acceptable bounds of the objective function over any interval subdomains. The last of the three is generally not specific to the dimension of problem (1). It is rather tied to the nature of the problem itself. Thus our new algorithm aims at easing the first two concerns only. Although interval algorithms can converge exponentially to the globally optimal objective function value [6], the number of subboxes to be saved and processed in an interval method could also increase exponentially with the dimension of the search domain. That raises memory requirement and slows down convergence. This could prevent any conventional interval method from becoming a practical choice for solving many large scale optimization problems. If the memory problem can be significantly alleviated, speed of convergence would be greatly improved. Consequently, interval methods would become more attractive than noninterval methods at least for the optimization problems where the function bounds are available.

Inspired by such observations, we have investigated some new strategies associated with the interval branch and bound methodology both theoretically and numerically. This paper reports one new version of the interval-based algorithm that shows improvement both in memory space usage and in overall speed of convergence. It is in fact essentially memoryless and yet still converges to a globally optimal solution in many cases. When it converges, it does so much more quickly than the standard interval method.

The rest of the paper is organized as follows. In Sect. 2, we review major features of the standard interval method. Our new algorithm is presented in Sect. 3 along with theoretical convergence results. Numerical testing results for a relatively large pool of examples are given in Sect. 4, followed by final comments and conclusions in Sect. 5. Finally 15 repeatedly used examples with variable dimensions are listed in the appendix.