GEOMECHANICS

INVESTIGATION INTO THE INFLUENCE OF STRESSES ON THE VELOCITIES OF ELASTIC WAVES IN THE VICINITY OF AN ELLIPTICAL MINE WORKING

V. L. Shkuratnik and G. V. Danilov

An analysis is performed for the relationship between the spatial distribution of the velocities of elastic wave propagation in the vicinity of an elliptical mine working and the external stress field.

Elliptical mine working, stress, elastic wave velocity

Mine working driving is attended with redistribution of the initial stress field in the surrounding rock mass. As a result, the following zones are formed at a distance from the mine working contour: zones of disturbed rocks with decreased bearing capacity, bearing pressure zones with the maximal stress level, and zones of natural stresses with no effect of the mine working on the rock stress-strain state [1]. Solution of some practical geomechanical problems needs knowing of boundaries of these zones, for example, when predicting stability of mine workings. At present, the appropriate information is experimentally obtained by the methods of ultrasonic sounding and logging. These methods make it possible to estimate distribution of the velocities $c_p$ of longitudinal elastic wave propagation depending on distance to the mine working contour. This distribution characterizes spatial change of stresses in the near-contour massif and may be used to reveal the boundaries of the above-mentioned zones [2].

At the same time, to interpret correctly results of acoustic measurements, it is required that conditionally “model” data on variation in $c_p$ as the stresses change, irrespectively of noise, are available. Also, these data are necessary for preliminary estimation of the effective parameters and modes of ultrasound measurements in a mass [3]. All these factors govern the importance of theoretical study of the problem on the effect of stresses on regularities of the velocity distribution of the elastic waves in the vicinity of a mine working.

To solve this problem, the authors have used a rather general model of a mine working with elliptical section (Fig. 1). Let the ellipse semi-axes $a$ and $b$ be much less then the mine working extent; then the elastic stress may be estimated in the approximation of a plane strain state [4].

Assume that at infinity, a stress $\sigma_0$ acts along the direction that makes an angle $\beta$ to the abscissa axis in the Cartesian coordinate system ($x, y$). With the Kolosov–Muskhelishvili method [5] used to solve two-dimensional problems of elasticity in an elliptical hole, we have that the complex potentials $\varphi$ and $\psi$ are in the form:

References:

Moscow State Mining University, E-mail: ftkp@mail.ru, Moscow, Russia. Translated from Fiziko-Tekhnicheskie Problemy Razrabotki Poleznykh Iskopаемых, No. 3, pp. 3-10, May-June, 2005. Original article submitted April 11, 2005.

1062-7391/05/4103-0195 ©2005 Springer Science + Business Media, Inc.
\[ \varphi(\zeta) = \frac{\sigma_0 R}{4} \left( \zeta + \frac{2e^{2i\beta} - m}{\zeta} \right); \]

\[ \psi(\zeta) = -\frac{\sigma_0 R}{2} \left[ e^{-2i\beta} \zeta + \frac{e^{2i\beta}}{m\zeta} - \frac{1 + m^2 (e^{2i\beta} - m)}{\zeta^2 - m} \right], \]

where \( R = (a + b) / 2 \), \( m = (a - b) / (a + b) \) (\( a > b \)), and \( \zeta = r e^{i\beta} \). The elliptical coordinates \( (\rho, \theta) \) are possible to be determined through the polar \((r, \varphi)\) or Cartesian coordinates of a current point (Fig. 1).

According to [4, 5], the stress tensor components \( \sigma_{\rho\rho}, \sigma_{\rho\theta}, \) and \( \sigma_{\theta\theta} \) are calculated as follows:

\[ \sigma_{\rho\rho} + \sigma_{\theta\theta} = 4 \text{Re} \Phi(\zeta), \]

\[ \sigma_{\theta\theta} - \sigma_{\rho\rho} + 2i\sigma_{\rho\theta} = \frac{2\zeta^2}{\rho^2 (1 - m\zeta^{-2})} \left[ \overline{\Phi}(\zeta) \Phi'(\zeta) + \omega'(\zeta) \Psi(\zeta) \right], \]

\[ \Phi(\zeta) = \frac{\varphi'(\zeta)}{\omega'(\zeta)}, \quad \Psi(\zeta) = \frac{\psi'(\zeta)}{\omega'(\zeta)}, \quad \omega(\zeta) = R(\zeta + m / \zeta), \]

where the bar symbol above the functions indicates the complex conjugation.

After summation of (1) and (2) at \( \beta = 0 \) \( (\sigma_0 = p) \) and at \( \beta = \pi / 2 \) \( (\sigma_0 = q) \), with (3) taken into account, we obtain:

\[ \sigma_{\theta\theta} - \sigma_{\rho\rho} + 2i\sigma_{\rho\theta} = \frac{2\zeta^2}{\rho^2 (1 - m\zeta^{-2})} \left[ \left( \overline{\Phi}(\zeta) \right) \Phi'(\zeta) + \left( 1 - \frac{m}{\zeta^2} \right) \Psi(\zeta) \right]; \]

\[ \Phi(\zeta) = \frac{1}{2} \left( A \frac{\zeta^2 + m}{\zeta^2 - m} + B \frac{2}{\zeta^2 - m} \right); \]

\[ \Psi(\zeta) = \frac{\zeta^2}{\zeta^2 - m} \left\{ A \frac{(1 + m^2)(\zeta^2 + m^2)}{(\zeta^2 - m)^2} + B \left[ 1 - \frac{1}{m\zeta^2 + m(\zeta^2 - m)^2} \right] \right\}, \]

where \( A = 0.5(p + q) \), \( B = 0.5(p - q) \); \( p, q \) are the principal components of the stresses at infinity along \( x \) and \( y \).

---

Fig. 1. Scheme for estimating distributions of stresses and velocities of longitudinal elastic wave propagation near mine working