EFFECTIVE ACTION OF CONCENTRATED FORCES ON A CRACK DIRECTED ALONG THE NORMAL TO THE ELASTIC HALF-PLANE BOUNDARY

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The stress intensity factor obtained at the tip of a straight crack directed along the normal to the elastic half-plane boundary is under consideration. The stress state is caused by the concentrated forces, symmetrical relative to the crack and applied to the half-plane boundaries. The efficiency conditions of such action are determined.

Crack, stress intensity factor, concentrated forces, half-plane

In [1] it is proposed to improve the percussive tool action efficiency by distributing an impact pulse into three components. Delivering time-delayed lateral blows at an angle to the central blow generates tensile stresses in the main crack formation zone and allows its enlarging at the same energy consumption. The lateral blows must be delivered at some distance from the point of the central blow application. The basis of such proposal are the results obtained in [2] for a disk crack in elastic space. It is established that the maximal stresses near the crack tip, produced by the action of the opposite-directed concentrated forces, are reached if the forces are applied somewhat apart from the crack edges.

The solution derived in [2] and the impact action model proposed in [1] differ mainly in the free boundary where the blow is delivered. In this study, it is considered how to act effectively on a crack by two symmetrical concentrated forces applied to the elastic half-plane boundary.

The problem is as follows. An elastic half-plane contains a straight crack of a length \( L \) that comes to the plane boundary along the normal to it. The concentrated forces are applied to the half-plane boundary at a point with coordinates \((b, 0)\). The tangential force \( T \) is directed along the \( Ox \) axis, and the normal force \( N \) is directed oppositely to the \( Oy \) axis. The coordinate system, crack location and concentrated forces are shown in Fig. 1a. The approximate, analytical formula to calculate a stress intensity factor (SIF) at the tip of a normal tension crack is obtained in [3]:

\[
K_\text{f}(\xi) = K_x(\xi)T + K_y(\xi)N,
\]

\[
\xi = \frac{b}{b+L}, \quad K_x(\xi) = \frac{(1-\xi^2)f_x(\xi)}{\sqrt{\pi L}}, \quad K_y(\xi) = \frac{(1-\xi^2)f_y(\xi)}{\sqrt{\pi L}},
\]

\[
f_x = 1.2943 + 0.0044\xi + 0.1289\xi^2 + 10.89\xi^3 - 22.14\xi^4 + 10.96\xi^5,
\]

\[
f_y = 0.8240 + 0.0637\xi - 0.8430\xi^2 + 15.41\xi^3 - 53.38\xi^4 + 59.74\xi^5 - 21.82\xi^6.
\]

This solution is in a good agreement with the known solutions for two symmetrical shear and normal forces applied at the crack origin [4, 5]. The calculation error of (1) is no more than 1%.

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Let two concentrated forces $P$, symmetrical relative to the crack, be applied to the half-plane boundary so that the force $(P_x, P_y)$ is applied at the point $(b, 0)$ and the force $(-P_x, P_y)$ at the point $(-b, 0)$ as is illustrated in Fig. 1b. In this case, $N = -P \cos \beta$, and $T = P \sin \beta$; thus, based on (1), we can write SIF for the normal tension crack as:

$$K_f(\xi, \beta) = 2P(K_x(\xi) \sin \beta - K_y(\xi) \cos \beta).$$

(2)

Figure 2 shows the isolines of the dimensionless function $K_f(\xi, \beta)/K_0$, where $K_0 = P/\sqrt{\pi L}$, calculated from (2). The angles $\beta$ and $\xi$ are expressed in degrees and are plotted in the axes of ordinates and abscissas, respectively. It is evident from Fig. 2 that SIF is considerably dependent, up to the sign, on the place and the angle of the concentrated force application to the crack. The function $K_f$ in the domain $\{0 \leq \beta \leq \pi, 0 \leq \xi \leq 1\}$ has an absolute maximum. It follows from the extremum condition $\partial K_f(\xi, \beta)/\partial \beta = 0$ and $\partial K_f(\xi, \beta)/\partial \xi = 0$ that:

$$\tan \beta = -\frac{K_x(\xi)}{K_y(\xi)} \quad \text{and} \quad \tan \beta = \frac{K_x'(\xi)}{K_y'(\xi)}.$$  

(3)