GEOMECHANICS

LIMIT LOADS IN THE PROBLEM OF INSTABILITY IN RIB PILLARS DURING AXISYMMETRICAL BULGING

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Determination of a limit load in the problem of instability of rib pillars in the course of axisymmetrical bulging involves three variants of the pillar’s pre-critical state: elasticity, perfect plasticity and post-limit deformation. The problem formulation is after Leibenzon – Ishlinskiy. The task is to find limit loads of a pillar with a pre-set dimension, such that the pillar instability is axisymmetrical.

Elasticity, plasticity, post-limit deformation, limit load

INTRODUCTION

Stability/instability of underground excavations is an object of numerous researches, for instance [1 – 10], where the instability is associated with a collapse of mine structures rather than with their deforming, while solid mechanics understands a body’s instability as deformation of this body (the loss of shape) when under a limit load [9 – 12]. For example, the loss of stability in a pillar may be referred to the pillar deforming when its initially rectangular cross section as in Fig. 1 transforms into a “barrel-shaped” profile (see Fig. 1b), with bulging sides, tensile stresses arise and rock pieces separate from the pillar.

This research makes an attempt to applying the approach and results from [11, 12] to estimation of pillar stability in compression under elastic, plastic and post-limit deformation with axial-symmetric bulging. Like [11, 12], it is suggested that there exists a basic stress-strain state and an additional stress-strain state associated with loss of stability in a pillar. The additional stress-strain state at the first moment of instability is assumed low, and values of the second order of vanishing are ignored. The basic stress-strain state is compression, while the additional state is complex additional load. And here we have to choose a mathematical model of a geo-medium.

Fig. 1. The present research subject, namely, loss of stability in a rib pillar
It is known that rocks differently resist tension and compression, and exhibit dilatancy in deformation. A geo-medium exposed to shear stresses can either increase in volume (loosening) or decrease (compaction), which depends on initial porosity and jointing of the medium. The math geo-model offered in [13, 14] included these and other features, but we think it suffices to use a simpler model here and analyze comprehensively geo-medium and its parameters in later studies.

MATHEMATICAL MODEL OF ELASTIC AND INELASTIC GEO-MEDIUM DEFORMATION

The present study relies upon some hypotheses: (1) deformation of a rib pillar is plane; (2) the average stress, \((\Delta \sigma_x + \Delta \sigma_y)/2\) any time of the deformation to the extent of the medium defragmentation, obeys the relation:

\[
\frac{\Delta \sigma_x + \Delta \sigma_y}{2} = k(\Delta \varepsilon_x + \Delta \varepsilon_y),
\]

where \(k\) is a constant of the material. In other words, volume of the medium under any loading conditions changes elastically [5].

And hypothesis (3) adverts to variation in deviator values. Let us consider a deviator plane for tensors of stresses, \(T_\sigma\), strains, \(T_\varepsilon\), and stress and strain increments \(T_{\Delta \sigma}, T_{\Delta \varepsilon}\). In the said plane the deviators of these tensors fit in with vectors \(\tau, \gamma, \Delta \tau, \Delta \gamma\) as in Fig. 2 [15].

We decompose vector \(\Delta \tau\) in line of vector \(\tau\) and normally to it (refer to Fig. 3). Additional loading in line of \(\tau\) is simple, and the \(\tau\)-normal loading is orthogonal [15]. We hypothesize (in concord with the plastic flow theory) that shear strain increment occurs in line of the simple additional loading and elastic strain increment follows the orthogonal direction, which means that:

\[
\Delta \gamma'' = \frac{\Delta \tau''}{2\mu_p} = \frac{\Delta \tau''}{2\mu},
\]

where \(2\mu_p\) is shear modulus in the curve of the maximum shear stress \(\tau = |\tau|\) versus the maximum shear \(\gamma = |\gamma|\) in Fig. 4.

The equations (2) in terms of the vectors \(\Delta \tau, \Delta \gamma\) can be written as:

\[
\frac{\Delta \sigma_x - \Delta \sigma_y}{2} = (\mu_p \cos^2 \theta + \mu \sin^2 \theta)(\Delta \varepsilon_x - \Delta \varepsilon_y) - (\mu - \mu_p) \sin 4\theta \cdot \Delta \varepsilon_{xy},
\]

\[
\Delta \tau_{xy} = -(\mu - \mu_p) \sin 4\theta \frac{\Delta \varepsilon_x - \Delta \varepsilon_y}{2} + 2(\mu_p \sin^2 \theta + \mu \cos^2 \theta) \Delta \varepsilon_{xy},
\]

where the module \(2\mu_p\) should be replaced by \(-2\mu_0\) (\(\mu_0 > 0\)) in case of post-limit deformation.