A Study of Local Solutions in Linear Bilevel Programming

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Abstract. In this paper, a linear bilevel programming problem (LBP) is considered. Local optimality in LBP is studied via two related problems (P) and P(M). Problem (P) is a one-level model obtained by replacing the innermost problem of LBP by its KKT conditions. Problem P(M) is a penalization of the complementarity constraints of (P) with a penalty parameter M. Characterizations of a (strict) local solution of LBP are derived. In particular, the concept of equilibrium point of P(M) is used to characterize the local optima of (P) and LBP.

Key Words. Bilevel linear programming, local optimization, exact penalty methods, equilibrium constraints.

1. Introduction

In this work, we consider the following linear bilevel program:

(LBP) max \( f_1(x, y) = c_1^T x + c_2^T y \)

s.t. \( x \geq 0, y \) solves \( max \ y \ f_2(x, y) = b^T y \)

s.t. \( A_1 x + A_2 y \leq a, y \geq 0 \)

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where $c_1, x \in \mathbb{R}^n_1$, $c_2, b, y \in \mathbb{R}^n_2$, $a \in \mathbb{R}^m$, $A_1 \in \mathbb{R}^{m \times n_1}$, $A_2 \in \mathbb{R}^{m \times n_2}$.

This problem has been studied extensively in the literature; see e.g. Refs. 1–6. The LBP with linear constraints in the first level has also been considered; see e.g. Ref. 7. We refer to Ref. 8 for a bibliographical survey and to Refs. 9–11 for more recent results on bilevel and multilevel programming.

Actually, this problem can be reformulated as a mathematical program with equilibrium constraints (MPEC), since the second-level problem can be replaced by a linear complementarity problem (Ref. 12). The two formulations are equivalent while considering global solutions, but the equivalence does not hold for local solutions. We are going to show that a local optimum of the MPEC formulation may not yield a local optimum of LBP.

Problem LBP belongs to the class of strongly NP-hard problems (Ref. 7). The main difficulties are due to its nonconvexity, which may result in an exponential number of local optima (Ref. 13). On the other hand, the design of algorithms has been made difficult due to the lack of computationally attractive theoretical results for the problem.

Our main aim is to derive necessary and sufficient conditions for local optimality in problem LBP. In particular, we state a characterization of a local optimum of LBP based on the notion of equilibrium point introduced in Ref. 6. This characterization is useful from a numerical point of view and may be used to devise local algorithms. Finding local solutions of nonconvex optimization problems is a meaningful deed itself. In addition, local procedures can be used within global algorithms. For problem LBP, this strategy is applied in Ref. 4, 5 for example.

The paper is organized as follows. Section 2 examines the auxiliary problems that will be used in the development. The approach follows that one presented in Ref. 6. In Section 3, we carry out the local analysis of LBP. We derive characterizations for its local and strict local solutions. The most computationally useful characterizations are based on the notion of equilibrium point, which is further explored in Section 4.

2. Preliminaries

In this section, we consider a problem related to LBP. The first auxiliary problem (P) is obtained by replacing the inner local problem of LBP by its KKT conditions. The second problem, P($\mathcal{M}$),