We study the problem of forced resonant vibrations and vibration heating of flexible viscoelastic beams with piezoelectric actuators and sensors. The viscoelastic behaviors of passive (without piezoelectric effect) and piezactive materials are described in terms of the instantaneous and complex moduli. To solve the nonlinear coupled problem of electroviscoelasticity and heat conduction, we use the method of quasilinearization together with the numerical methods of discrete orthogonalization and finite differences. We also study the influence of boundary conditions and geometric nonlinearities on the dynamic characteristics and electric parameters of a sensor and on the temperature of vibration heating of the flexible beam. For the active damping of the beams, we propose a procedure of finding the actuator parameter according to the sensor parameter for the unknown external load.

Introduction

The methods of active damping of forced vibrations of thin-walled elements with the help of piezoelectric inclusions are now extensively developed [16–19, 21]. One of the most widespread methods of active damping is based on the application of piezactive layers to the surface of a passive carrying substrate. In this case, one layer plays the role of a sensor and the second layer serves as an actuator. The problems of forced vibrations and vibration heating of viscoelastic thin-walled elements and their monitoring with the help of piezoelectric sensors and actuators are investigated in [3, 7–15, 17, 20] and other works. If a thin-walled element operates under the conditions of intense harmonic loads and its deflections become comparable with its thickness, then it is necessary to take into account the effects of physical and geometric nonlinearity in the problems of analysis of the thermomechanical behavior of elements of this kind.

The construction of the electrothermomechanical models of thin-walled elements made of viscoelastic passive (without piezoelectric effect) and piezactive components with regard for physical and geometric nonlinearities and the solutions of several problems can be found in [3, 6, 7, 10, 12]. In particular, the problems of forced vibrations and vibration heating of simply supported viscoelastic beams with piezoelectric actuators are studied in [3] and [14] with regard for physical and geometric nonlinearities, respectively.

In the present work, we consider the problem of forced vibrations and dissipative heating of viscoelastic flexible beams with piezoelectric sensors and actuators with different boundary conditions. We investigate the influence of the mechanical conditions of fixing and geometric nonlinearities on the dynamic and temperature characteristics of the system under study, electric parameters of the sensor, feedback factor, and the critical loads under which the temperature of vibration heating reaches the Curie point in the piezoelectric material and either the temperature of softening or melting point in the passive material.
1. Statement of the Problem

Consider a three-layer beam of length $\ell$ and width $b$. The middle layer of thickness $h_0$ is made of an isotropic passive (without piezoelectric effect) viscoelastic material and the outer layers of thickness $h_1$ are made of viscoelastic piezoelectric ceramics with thickness polarization in the opposite directions. The layers are rigidly fastened to each other. The beam is referred to a Cartesian coordinate system $Oxyz$ so that $0 \leq x \leq \ell$, $|y| \leq b/2$, and $|z| \leq H/2$ (where $H = h_0 + 2h_1$).

We assume that one of the piezolayers ($z \leq -h_0/2$) plays the role of an actuator whose polarization is characterized by the piezoelectric modulus $d_{31} = d_{31}$ and the second layer ($z \geq h_0/2$) serves as a sensor with a piezoelectric modulus $d_{31} = -d_{31}$. The outer ($z = \pm H/2$) and inner ($z = \pm h_0/2$) surfaces of the piezoelectric layers are covered with electrodes. Here, the inner electrodes are kept at an electric potential $\varphi(\pm h_0/2) = 0$. The outer surfaces with electrodes are split into separate domains by infinitely thin cuts with coordinates $x_0$ and $x_1$. A surface pressure $q_z = q_0 + q' \cos \omega t$ with a nearly resonant circular frequency $\omega$ of its harmonic component acts upon the beam. To the electrodes of the actuator with area $s_x = b\Delta x$ (where $\Delta x = x_1 - x_0$), we apply a voltage

$$\varphi_1(-h_0/2) - \varphi_1(-H/2) = \text{Re} (V_a e^{i\omega t})$$

with the same frequency as the mechanical load whose action is compensated or amplified depending on the amplitude and phase of the supplied voltage. In the case of disconnected sensor electrodes, the following electrostatic condition is satisfied on the surfaces with electrodes [12]:

$$\iiint_s 2D_z = 0,$$  \hspace{1cm} (1)

where $2D_z$ is the normal component of electric induction in the piezoelectric layer of the sensor.

As a result of deformation of the beam, on the disconnected electrodes of the sensor, we observe the appearance of a potential difference with unknown amplitude $V_2 = \varphi_2(H/2) - \varphi_2(h_0/2)$, which should be found either numerically or experimentally.

In modeling the electrothermomechanical behavior of the three-layer flexible beams, we suppose that the entire stack of layers obeys the hypotheses of flat cross sections [1] for the mechanical quantities and the adequate assumptions concerning the electric variables [6, 14]. As follows from these assumptions, the components of electric induction $mD_z = mC = \text{const}$ ($m = 1, 2$) are constant over the thickness of piezoelectric layers. The temperature of dissipative heating of the beam is set constant over the thickness of the stack of layers. Moreover, we assume that the strains are low but the deflections of the beam are such that the squared angles of rotation should be preserved in the kinematic equations. Thus, the equations of motion are also nonlinear. The viscoelastic properties of materials of the layers are described by the integral operators [4] that can be reduced to the multiplication of complex variables for the harmonic processes of deformation [5]:

$$B \ast f = (B' + iB'') (f' + if'').$$  \hspace{1cm} (2)

In view of the accepted hypotheses, the three-dimensional relations for viscoelastic piezoelectric ceramics polarized along the $Oz$-axis [5, 6] take the form